

TRANSIENT GROUND WATER HYDRAULICS

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Chapter 8Parallel drains

In areas where natural drainage is inadequate, irrigation will cause the water table to rise progressively until the land becomes water logged. To improve the drainage, parallel drains can be installed. These may take the form of drainage canals or of tile drains laid in a trench and back filled. The latter arrangement has the advantage that the installation of drains does not take any land out of production.

First approximation solution

A solution of the differential equation 2-2

$$\alpha \frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t}$$

Subject to the conditions

$$h = 0 \quad \text{when } x = 0 \quad \text{for } t > 0$$

$$h = 0 \quad \text{when } x = L \quad \text{for } t > 0$$

$$h = H \quad \text{when } t = 0 \quad \text{for } 0 < x < L$$

is

$$h = \frac{4H}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{e^{-n^2 \pi^2 \left(\frac{\alpha t}{L^2}\right)}}{n} \sin \left(\frac{n\pi x}{L}\right) \quad (8-1)$$

A cross section normal to the drains is shown in figure 8-1. When $x = \frac{L}{2}$ this expression takes the form

$$h_c = \frac{4H}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{e^{-n^2 \pi^2 \left(\frac{\alpha t}{L^2}\right)}}{n} \sin \left(\frac{n\pi}{2}\right) \quad (8-2)$$

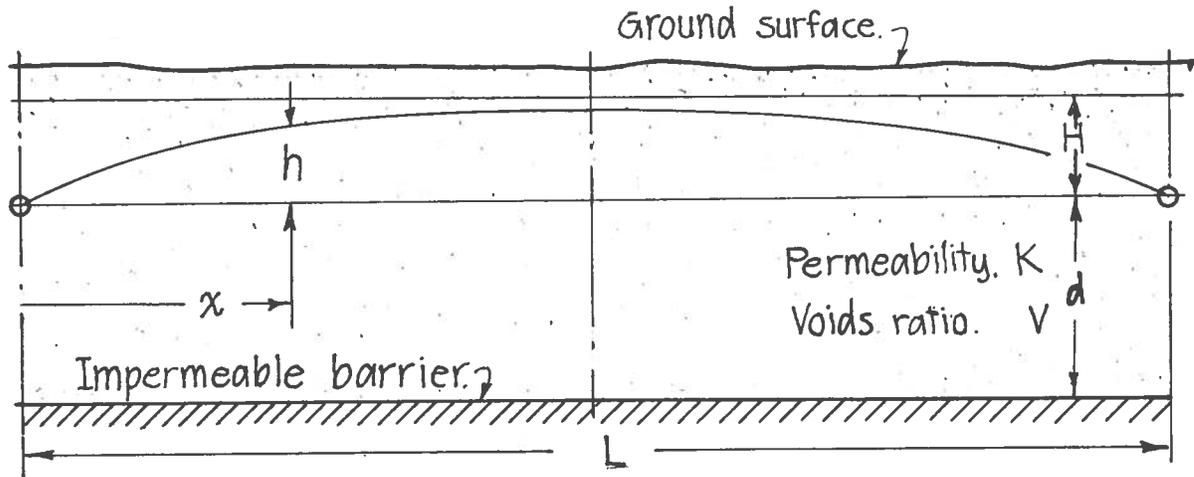


Fig. 8-1 Cross section normal to the line of drains.

Values of this function are listed in Table 10. These values find use in the technically important task of selecting a drain spacing to fit a specified set of field conditions. This is because the most difficult point to drain is midway between the drains. If this point can be drained then every other point will be drained also.

The flow to a drain from one side is

$$Kd \left(\frac{\partial h}{\partial x} \right)_{x=0} = \frac{4KdH}{L} \sum_{n=1,3,5,\dots}^{\infty} e^{-n^2 \pi^2 \left(\frac{\alpha t}{L^2} \right)} \quad (8-3)$$

It may be noted in passing that this function has a singularity at $t = 0$. This value must be disregarded as the infinite gradient obtained from the above formula conflicts with the requirement that, for validity, the gradient must be small compared to unity. It will be shown later that there is a local resistance due to the convergence of the flow approaching a tile drain which limits the flow rate to a finite value.

Another quantity of importance is the fractional part of the drainable volume remaining at the time t . This is obtained from the relation

$$p = \frac{1}{HL} \int_0^L h \, dx = \frac{8}{\pi^2} \sum_{n=1,3,5,\dots}^{n=\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n^2} \quad (8-4)$$

Values of p are obtainable from Table 11.

Application of Werner's method

The first approximation solution is also a solution of Werner's differential equation. The boundary and initial conditions are also appropriate but since h_2 is measured from the barrier the case represented is one where the drain is on the barrier. Then

$$\frac{h_2}{H} = \sqrt{\frac{4}{\pi} \sum_{n=1,3,5,\dots}^{n=\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n} \sin\left(\frac{n\pi x}{L}\right)} \quad (8-5)$$

The development of Boussinesq

A transient state drainage treatment has been contributed by J. Boussinesq (Boussinesq 1904). He used concepts very similar to those employed by Dupuit in that he assumed the surface gradient to apply throughout the saturated depth. In our notation the condition of continuity would take the form

$$\frac{\partial}{\partial x} \left(Kh_2 \frac{\partial h_2}{\partial x} \right) = V \frac{\partial h_2}{\partial t} \quad (8-6)$$

The development applies where the drains are on the barrier at a distance L apart. The distance x is measured from one of the drains toward the other. The drainable depth h_2 has the value H at $x = \frac{L}{2}$ when the time $t = 0$.

Let

$$U = \frac{h_2}{H}$$

$$\xi = \frac{x}{L}$$

$$\eta = \frac{KH}{VL^2} t$$

Then the differential equation takes the form

$$\frac{\partial}{\partial \xi} \left(U \frac{\partial U}{\partial \xi} \right) = \frac{\partial U}{\partial \eta} \quad (8-7)$$

A possible type of solution is

$$U = WY$$

Where W is a function of ξ only and Y is a function of η only.

Substitution of this product into the differential equation permits a separation of the variables and yields two ordinary differential equations one in W and the other in Y . In the case of Y the relation is

$$\frac{1}{Y^2} \frac{dY}{d\eta} = -C$$

And a solution satisfying the conditions $Y = 1$ when $\eta = 0$ is

$$Y = \frac{1}{C\eta + 1}$$

The differential equation for W is of the nonlinear form

$$\frac{d^2W}{d\xi^2} + \frac{1}{W} \left(\frac{dW}{d\xi} \right)^2 = -C$$

This can be reduced to a first order differential equation by the substitutions

$$\frac{dW}{d\xi} = p \qquad \frac{d^2W}{d\xi^2} = p \frac{dp}{dW}$$

After substitution the above differential equation becomes

$$p \frac{dp}{dW} + \frac{p^2}{W} = -C$$

Where it may be noted that W has now become the independent variable. A further substitution

$$v = p^2 \qquad \frac{dv}{dW} = 2p \frac{dp}{dW}$$

reduces it to the linear form

$$\frac{dv}{dW} + \frac{2v}{W} = -2C$$

A solution is

$$vW^2 = -\frac{2CW^3}{3} + C_2$$

or

$$p^2 = -\frac{2CW}{3} + \frac{C_2}{W^2}$$

If $p = 0$ when $W = 1$ $C_2 = \frac{2C}{3}$ and

$$\frac{W}{\sqrt{1-W^3}} \frac{dW}{d\xi} = \sqrt{\frac{2C}{3}}$$

By integration subject to the condition that $W = 0$ when $\xi = 0$.

$$\int_0^W \frac{WdW}{\sqrt{1-W^3}} = \sqrt{\frac{2C}{3}} \xi \quad (8-8)$$

When $\xi = \frac{1}{2}$ $W = 1$ then

$$\int_0^1 \frac{WdW}{\sqrt{1-W^3}} = \sqrt{\frac{C}{6}}$$

The integral of this relation can be evaluated with the aid of the Beta and Gamma functions (Osgood 1933, p. 485). The evaluation is

$$\int_0^1 \frac{WdW}{\sqrt{1-W^3}} = \frac{\Gamma(\frac{1}{2}) \Gamma(\frac{2}{3})}{3\Gamma(\frac{7}{6})} = 0.86237$$

then

$$C = (6) (0.86237)^2 = 4.46209 \quad \sqrt{\frac{2C}{3}} = 1.72474$$

The following table is reproduced through the courtesy of Mr. W. T. Moody.

ξ	W
0.0	0.0
0.005	0.412
0.10	0.575
0.15	0.692
0.20	0.782
0.25	0.853
0.30	0.908
0.35	0.949
0.40	0.978
0.45	0.994
0.50	1.000

Finally

$$\frac{h}{H} = \frac{W}{4.46 \left(\frac{KHt}{VL^2} \right) + 1} \quad (8-9)$$

This development is of value because it gives indications as to what drainage performance is to be expected when the drains are near the barrier. The first approximation solution gives little guidance in such cases. It may be noted that the initial condition of a uniform drainable depth is not met by the Boussinesq development. It is interesting to note also that the pattern of decrease is not of a descending exponential type but here takes an algebraic form.

The Method of Brooks

A second approximation solution which remains valid when the drainable depth is not negligibly small compared to the saturated depth below the drains was obtained by Brooks 1963 by application of the Pioncare, Lighthill, Kuo method. Good results were obtained where drainable depth were as great as the saturated depth below the drains ($H/d = 1.0$). He also developed a second approximation of the type described by Haushild and Kruse, 1962. This formula can be expressed in our notation as

$$h_1 = -D_a + \sqrt{D_a^2 + 2D_a h_0 + \left(\frac{H}{2}\right)^2} \quad (8-10)$$

Where $D_a = \left(d + \frac{H}{2}\right)$ and h_0 comes from the first approximation. It is obtained by computing the drainable depths on a nonlinear basis based upon the flows obtained from the first approximation. The original paper may need to be consulted. His origin is placed midway of the original drainable depth.

The Method of Dumm, Tapp and Moody

This procedure was developed at the U.S. Bureau of Reclamation* to provide an orderly approach to the problem of determining drain spacings. It was recognized that there would be applications where the drainable depth would not be small when compared to the barrier depth. It was also understood that where the drainable depth is not small, in the above sense, the basic differential equation would be nonlinear in form which would mean that the principle of superposition would not apply since the sum of two solutions is then not a solution. As a consequence of this concept the superposition of uniform increments of drainable depth originating in uniform irrigations was abandoned in favor of a drainage pattern representing observed configurations after a number of irrigations had been made. Computations of drainage progress were begun anew with each irrigation. The increment of added drainable depth being added to the depth obtained from the preceding calculation. Computations are made for the point midway between drains. The pattern chosen to represent the data obtained from field observations is

$$h = 8H \left(\frac{x}{L} - \frac{3x^2}{L^2} + \frac{4x^3}{L^3} - \frac{2x^4}{L^4} \right) \quad (8-11)$$

A first approximation solution having this initial configuration is given as

$$h_1 = \frac{192H}{\pi^5} \sum_{m=0}^{m=\infty} \frac{[(2m+1)^2 \pi^2 - 8] e^{-\frac{(2m+1)^2 \pi^2 (\alpha t)}{L^2}}}{(2m+1)^5} \sin \frac{(2m+1)\pi x}{L} \quad (8-12)$$

At the point midway between drains this takes the form

* See also USBR Eng. Monograph 31, 1966.

$$\left(\frac{h_1}{H}\right) \frac{L}{2} = \frac{192}{\pi^3} \sum_{n=1,3,5,\dots}^{n=\infty} \frac{(-1)^{\frac{n-1}{2}} (n^2 - \frac{8}{\pi^2}) e^{-n^2 \pi^2 (\frac{\alpha t}{L^2})}}{n^5} \quad (8-13)$$

The initial drainable depth at the point midway between drains will here be represented by H_m . The quantity D_m is expressible in the notation of this volume as $D_m = (d + H_m)$ and the aquifer constant is, correspondingly, $\alpha_m = \frac{KD_m}{V}$. The quantity q_m represents the rate of discharge to unit length of drains from the space between two drains, or, if a single drain is considered, it represents the flow to unit length of drain, from both sides. The quantity W_m represents the total flow to a unit length of drain from both sides. The quantity m is given by the relation $m = \left(\frac{H_c}{d + H_c}\right)$. The quantity h_m represents the drainable depth midway between drains at the time t .

For the case where the drains are on the barrier Boussinesq's solution is used.

The intermediate case was treated by comparison with field tests. It was found that the first approximation formula could be used providing an aquifer constant of the form:

$$\alpha = \frac{K D_a}{V} \quad \text{was used, where} \quad D_a = \left(d + \frac{H_c}{2}\right).$$

Moody's development

A development by Moody 1966 does away with the need to make such a choice. He used a computer to solve the nonlinear differential equation for a series of drain positions ranging from a location near the water table to a location on the barrier. He produced a table giving the maximum water table height, the

discharge to the drains and the volume of water removed. This is in dimensionless form and covers the entire range of possible drain positions between the water table and the barrier. The table is reproduced here as Table 8-1 through the courtesy of Mr. W. T. Moody and of the U. S. Bureau of Reclamation.*

The procedures described have been presented to the Profession in a series of papers and much comment has been received both favorable and unfavorable. The authors have made changes to meet the unfavorable comments and have correlated their methods with field data from Australia, Canada and the United States. The method has also been correlated with the results of laboratory studies. There is also accumulating the experience with field installations for which drain spacings have been selected by application of the method. The experience with such installations is understood to have been satisfactory. So far as this writer is aware, this method is the most carefully worked out, has received the most searching scrutiny, and has been more extensively tested against laboratory and field data than any method proposed for determining the spacing of drains which will, on the one hand, provide satisfactory drainage and on the other hand avoid the excessive expenditures incurred when the drains are spaced closer than necessary. Some valuable by-products were obtained in the period of development, which has now covered 12 years and a study of the original papers is therefore recommended to those who become seriously involved in the task of selecting drain spacings. One of these is the need to account for the local resistance to flow near the drain. This problem will be dealt with later in the text.

* From unpublished USBR data. For a Graphical Presentation see ASCE Paper No. 4835, by William T. Moody, on "Nonlinear Differential Equation of Drain Spacing," Journal of the Irrigation and Drainage Division, June 1966, pp. 1-9, inclusive.

Table 8-1

Values of Dimensionless Parameters for Water Surface Height (h_m/H_m) Flow Rate (q_{mL}/KD_mH_m) and Volume Drained (W_m/VLH_m) for Given Parameters of Time ($\alpha_m t/L^2$). For notation see page 98.

$\frac{\alpha_m t}{L^2}$	$m = 1.0$			$m = 0.9$			$m = 0.8$			$m = 0.7$			$m = 0.6$		
	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_mH_m}$	$\frac{W_m}{VLH_m}$												
0.	1.	1.9746	0.	1.	3.2249	0.	1.	4.4752	0.	1.	5.7254	0.	1.	6.9757	0.
0.0002	1.0000	2.0845	0.0004	1.0000	3.3219	0.0007	1.0000	4.5506	0.0009	1.0000	5.7710	0.0012	1.0000	6.9831	0.0014
0.0004	0.9999	2.1915	0.0008	0.9999	3.4138	0.0013	0.9999	4.6199	0.0018	0.9999	5.8103	0.0023	0.9999	6.9852	0.0028
0.0006	0.9999	2.2928	0.0013	0.9999	3.4980	0.0020	0.9999	4.6809	0.0028	0.9999	5.8419	0.0035	0.9999	6.9816	0.0042
0.0008	0.9998	2.3898	0.0018	0.9998	3.5763	0.0027	0.9998	4.7355	0.0037	0.9998	5.8679	0.0047	0.9998	6.9745	0.0056
0.0010	0.9997	2.4815	0.0023	0.9997	3.6482	0.0035	0.9997	4.7835	0.0047	0.9997	5.8882	0.0058	0.9997	6.9634	0.0070
0.0012	0.9996	2.5687	0.0028	0.9996	3.7144	0.0042	0.9996	4.8259	0.0056	0.9996	5.9040	0.0070	0.9996	6.9496	0.0084
0.0014	0.9995	2.6508	0.0033	0.9995	3.7752	0.0050	0.9995	4.8632	0.0066	0.9995	5.9155	0.0082	0.9995	6.9331	0.0098
0.0016	0.9994	2.7282	0.0038	0.9994	3.8310	0.0057	0.9994	4.8959	0.0076	0.9994	5.9234	0.0094	0.9994	6.9146	0.0112
0.0018	0.9993	2.8011	0.0044	0.9992	3.8820	0.0065	0.9992	4.9242	0.0085	0.9992	5.9280	0.0106	0.9992	6.8942	0.0125
0.0020	0.9991	2.8695	0.0049	0.9991	3.9286	0.0073	0.9991	4.9487	0.0095	0.9991	5.9297	0.0118	0.9991	6.8724	0.0139
0.0040	0.9968	3.3459	0.0112	0.9967	4.2107	0.0155	0.9967	5.0459	0.0196	0.9967	5.8462	0.0236	0.9967	6.6097	0.0274
0.0060	0.9932	3.5618	0.0182	0.9931	4.2880	0.0240	0.9931	4.9969	0.0296	0.9930	5.6788	0.0351	0.9930	6.3289	0.0403
0.0080	0.9885	3.6382	0.0254	0.9884	4.2695	0.0325	0.9883	4.8910	0.0395	0.9882	5.4908	0.0463	0.9880	6.0630	0.0527
0.0100	0.9831	3.6422	0.0327	0.9829	4.2061	0.0410	0.9826	4.7644	0.0492	0.9824	5.3042	0.0570	0.9821	5.8192	0.0646
0.0120	0.9770	3.6084	0.0399	0.9766	4.1224	0.0494	0.9762	4.6328	0.0586	0.9758	5.1267	0.0675	0.9753	5.5979	0.0760
0.0140	0.9706	3.5551	0.0471	0.9699	4.0306	0.0575	0.9693	4.5034	0.0677	0.9685	4.9609	0.0776	0.9678	5.3971	0.0870
0.0160	0.9638	3.4921	0.0541	0.9628	3.9368	0.0655	0.9619	4.3791	0.0766	0.9608	4.8070	0.0873	0.9597	5.2146	0.0976
0.0180	0.9567	3.4248	0.0610	0.9555	3.8441	0.0733	0.9542	4.2612	0.0852	0.9528	4.6644	0.0968	0.9512	5.0481	0.1079
0.0200	0.9496	3.3563	0.0678	0.9480	3.7542	0.0809	0.9462	4.1499	0.0936	0.9444	4.5321	0.1060	0.9424	4.8954	0.1178
0.0400	0.8779	2.7630	0.1287	0.8711	3.0466	0.1483	0.8637	3.3255	0.1677	0.8557	3.5911	0.1863	0.8472	3.8409	0.2041
0.0600	0.8133	2.3404	0.1795	0.8006	2.5708	0.2042	0.7868	2.7936	0.2285	0.7721	3.0019	0.2519	0.7566	3.1944	0.2741
0.0800	0.7569	2.0182	0.2229	0.7387	2.2134	0.2519	0.7191	2.3981	0.2803	0.6984	2.5663	0.3074	0.6766	2.7178	0.3330

Table 8-1--Continued

$\alpha_m t$ $\frac{L^2}{L^2}$	$m = 1.0$			$m = 0.9$			$m = 0.8$			$m = 0.7$			$m = 0.6$		
	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VIH_m}$	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VLH_m}$
0.1000	0.7076	1.7613	0.2607	0.6845	1.9296	0.2933	0.6598	2.0848	0.3250	0.6338	2.2214	0.3551	0.6069	2.3401	0.3834
0.1200	0.6643	1.5514	0.2937	0.6369	1.6982	0.3295	0.6077	1.8291	0.3641	0.5773	1.9398	0.3967	0.5460	2.0315	0.4270
0.1400	0.6260	1.3772	0.3230	0.5947	1.5061	0.3614	0.5617	1.6168	0.3985	0.5274	1.7059	0.4330	0.4926	1.7750	0.4650
0.1600	0.5918	1.2309	0.3490	0.5572	1.3446	0.3899	0.5207	1.4384	0.4290	0.4833	1.5093	0.4651	0.4455	1.5595	0.4983
0.1800	0.5612	1.1067	0.3723	0.5235	1.2076	0.4154	0.4841	1.2869	0.4562	0.4439	1.3423	0.4936	0.4038	1.3767	0.5276
0.2000	0.5336	1.0004	0.3934	0.4932	1.0903	0.4383	0.4511	1.1571	0.4806	0.4087	1.1994	0.5190	0.3668	1.2205	0.5536
0.4000	0.3576	0.4494	0.5275	0.3006	0.4811	0.5835	0.2450	0.4839	0.6314	0.1941	0.4637	0.6705	0.1501	0.4274	0.7019
0.6000	0.2689	0.2541	0.5951	0.2046	0.2650	0.6550	0.1467	0.2481	0.7011	0.1000	0.2146	0.7345	0.0654	0.1738	0.7578
0.8000	0.2155	0.1632	0.6358	0.1477	0.1644	0.6969	0.0923	0.1413	0.7388	0.0534	0.1081	0.7654	0.0292	0.0752	0.7813
1.0000	0.1798	0.1136	0.6630	0.1106	0.1099	0.7238	0.0596	0.0855	0.7610	0.0290	0.0569	0.7813	0.0132	0.0335	0.7916
1.2000	0.1542	0.0836	0.6825	0.0848	0.0771	0.7422	0.0391	0.0537	0.7746	0.0159	0.0306	0.7898	0.0060	0.0151	0.7962
1.4000	0.1350	0.0641	0.6971	0.0661	0.0561	0.7554	0.0259	0.0345	0.7833	0.0088	0.0167	0.7944	0.0027	0.0068	0.7983
1.6000	0.1200	0.0507	0.7085	0.0522	0.0419	0.7651	0.0173	0.0226	0.7889	0.0048	0.0092	0.7969	0.0012	0.0031	0.7992
1.8000	0.1081	0.0411	0.7176	0.0415	0.0319	0.7725	0.0116	0.0149	0.7926	0.0027	0.0051	0.7983	0.0006	0.0014	0.7996
2.0000	0.0983	0.0340	0.7251	0.0333	0.0246	0.7781	0.0078	0.0099	0.7950	0.0015	0.0028	0.7991	0.0003	0.0006	0.7998

Table 8-1--Continued

$\frac{\Delta C_{mt}}{L^2}$	$m = 0.5$					$m = 0.4$					$m = 0.3$					$m = 0.2$					$m = 0.1$				
	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_L}{KDH}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_L}{KDH}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KD_m H_m}$	$\frac{W_m}{VLH_m}$	
0.	1.	8.2260	0.	1.	9.4762	0.	1.	10.7265	0.	1.	11.9768	0.	1.	13.2271	0.	1.	14.5274	0.	1.	15.8277	0.	1.	17.1280	0.	
0.0002	1.0000	8.1873	0.0017	1.0000	9.3837	0.0019	1.0000	10.5724	0.0021	1.0000	11.7538	0.0024	1.0000	12.9280	0.0026	1.0000	14.1032	0.0028	1.0000	15.2785	0.0030	1.0000	16.4537	0.0032	
0.0004	0.9999	8.1452	0.0033	0.9999	9.2905	0.0038	0.9999	10.4217	0.0042	0.9999	11.5389	0.0047	0.9999	12.6428	0.0052	0.9999	13.8181	0.0056	0.9999	14.9934	0.0060	0.9999	16.1687	0.0064	
0.0006	0.9999	8.1006	0.0049	0.9999	9.1994	0.0056	0.9999	10.2786	0.0063	0.9999	11.3389	0.0070	0.9999	12.4542	0.0077	0.9999	13.6295	0.0084	0.9999	14.8048	0.0090	0.9999	16.0101	0.0096	
0.0008	0.9998	8.0559	0.0065	0.9998	9.1132	0.0074	0.9998	10.1470	0.0084	0.9998	11.1584	0.0092	0.9998	12.2837	0.0101	0.9998	13.4590	0.0108	0.9998	14.6343	0.0116	0.9998	15.9096	0.0124	
0.0010	0.9997	8.0100	0.0081	0.9997	9.0289	0.0093	0.9997	10.0214	0.0104	0.9997	10.9887	0.0115	0.9997	12.1539	0.0125	0.9997	13.3292	0.0128	0.9997	14.5045	0.0132	0.9997	15.8351	0.0140	
0.0012	0.9996	7.9640	0.0097	0.9996	8.9484	0.0111	0.9996	9.9043	0.0124	0.9996	10.8331	0.0136	0.9996	12.0296	0.0149	0.9996	13.2049	0.0148	0.9996	14.3802	0.0152	0.9996	15.7356	0.0160	
0.0014	0.9995	7.9174	0.0113	0.9995	8.8698	0.0128	0.9995	9.7920	0.0143	0.9995	10.6858	0.0158	0.9995	11.8611	0.0172	0.9995	13.0364	0.0163	0.9995	14.2117	0.0168	0.9995	15.6624	0.0180	
0.0016	0.9994	7.8710	0.0129	0.9994	8.7943	0.0146	0.9994	9.6863	0.0163	0.9994	10.5489	0.0179	0.9994	11.7172	0.0195	0.9994	12.8925	0.0182	0.9994	14.0678	0.0190	0.9994	15.5435	0.0200	
0.0018	0.9992	7.8245	0.0145	0.9992	8.7206	0.0164	0.9992	9.5845	0.0182	0.9992	10.4185	0.0200	0.9992	11.5938	0.0218	0.9992	12.7691	0.0200	0.9992	13.9444	0.0208	0.9992	15.4191	0.0224	
0.0020	0.9991	7.7784	0.0160	0.9991	8.6494	0.0181	0.9991	9.4879	0.0201	0.9991	10.2961	0.0221	0.9991	11.4714	0.0240	0.9991	12.6467	0.0221	0.9991	13.8170	0.0228	0.9991	15.2727	0.0248	
0.0040	0.9967	7.3369	0.0311	0.9967	8.0298	0.0347	0.9967	8.6914	0.0382	0.9967	9.3247	0.0416	0.9967	10.4969	0.0449	0.9967	11.6722	0.0416	0.9967	12.8475	0.0424	0.9967	14.9932	0.0460	
0.0060	0.9929	6.9468	0.0454	0.9929	7.5339	0.0503	0.9928	8.0932	0.0550	0.9928	8.6277	0.0596	0.9928	9.8030	0.0640	0.9927	11.0083	0.0596	0.9927	12.1836	0.0604	0.9927	14.4391	0.0660	
0.0080	0.9879	6.6062	0.0590	0.9878	7.1220	0.0649	0.9876	7.6129	0.0707	0.9875	8.0819	0.0763	0.9875	9.2972	0.0816	0.9873	10.5025	0.0763	0.9873	11.6778	0.0780	0.9873	14.0883	0.0860	
0.0100	0.9818	6.3079	0.0719	0.9815	6.7716	0.0788	0.9812	7.2129	0.0855	0.9809	7.6345	0.0920	0.9809	8.5180	0.0982	0.9805	9.7233	0.0920	0.9805	10.8988	0.0948	0.9805	13.4993	0.1020	
0.0120	0.9748	6.0447	0.0842	0.9742	6.4686	0.0921	0.9737	6.8718	0.0996	0.9731	7.2570	0.1068	0.9731	8.1723	0.1138	0.9725	9.3778	0.1068	0.9725	10.5534	0.1104	0.9725	13.1739	0.1180	
0.0140	0.9670	5.8106	0.0961	0.9661	6.2027	0.1047	0.9652	6.5758	0.1130	0.9643	6.9321	0.1210	0.9643	7.8723	0.1287	0.9633	8.9730	0.1210	0.9633	10.1586	0.1248	0.9633	12.8591	0.1360	
0.0160	0.9586	5.6008	0.1075	0.9573	5.9669	0.1169	0.9560	6.3152	0.1259	0.9546	6.6480	0.1346	0.9546	7.7178	0.1430	0.9530	8.7583	0.1346	0.9530	9.9436	0.1392	0.9530	12.6341	0.1500	
0.0180	0.9496	5.4113	0.1185	0.9479	5.7558	0.1286	0.9460	6.0834	0.1383	0.9441	6.3964	0.1476	0.9441	7.5188	0.1530	0.9420	8.6241	0.1476	0.9420	9.8092	0.1588	0.9420	12.4891	0.1640	
0.0200	0.9403	5.2392	0.1291	0.9380	5.5651	0.1399	0.9356	5.8752	0.1503	0.9330	6.1713	0.1602	0.9330	7.3103	0.1698	0.9302	8.4530	0.1602	0.9302	9.7436	0.1756	0.9302	12.3441	0.1760	
0.0400	0.8382	4.0756	0.2210	0.8285	4.2969	0.2371	0.8184	4.5066	0.2525	0.8077	4.7061	0.2672	0.8077	5.5113	0.2814	0.7964	6.5160	0.2672	0.7964	7.7407	0.2872	0.7964	10.6011	0.3040	
0.0600	0.7402	3.3725	0.2956	0.7231	3.5380	0.3150	0.7053	3.6920	0.3340	0.6869	3.8357	0.3521	0.6869	4.4403	0.3695	0.6679	5.4450	0.3521	0.6679	6.6701	0.3800	0.6679	9.6901	0.4000	
0.0800	0.6540	2.8542	0.3571	0.6307	2.9771	0.3799	0.6068	3.0875	0.4015	0.5825	3.1860	0.4221	0.5825	3.8357	0.4417	0.5579	4.7403	0.4221	0.5579	5.9701	0.4600	0.5579	9.0801	0.4800	

Table 8-1--Continued

$\frac{\alpha_m t}{L^2}$	$m = 0.5$			$m = 0.4$			$m = 0.3$			$m = 0.2$			$m = 0.1$		
	$\frac{h_m}{H_m}$	$\frac{q_{mL}}{KDm^2H_m}$	$\frac{W_m}{VLH_m}$												
0.1000	0.5792	2.4427	0.4099	0.5510	2.5306	0.4348	0.5225	2.6046	0.4583	0.4940	2.6652	0.4804	0.4658	2.7128	0.5014
0.1200	0.5142	2.1061	0.4553	0.4823	2.1649	0.4817	0.4506	2.2087	0.5063	0.4194	2.2381	0.5293	0.3890	2.2537	0.5509
0.1400	0.4576	1.8263	0.4945	0.4230	1.8611	0.5218	0.3891	1.8803	0.5471	0.3564	1.8846	0.5704	0.3251	1.8750	0.5920
0.1600	0.4081	1.5914	0.5287	0.3716	1.6065	0.5564	0.3365	1.6058	0.5819	0.3031	1.5904	0.6051	0.2717	1.5617	0.6263
0.1800	0.3646	1.3926	0.5584	0.3269	1.3915	0.5864	0.2912	1.3750	0.6116	0.2579	1.3446	0.6344	0.2272	1.3020	0.6549
0.2000	0.3263	1.2231	0.5846	0.2880	1.2090	0.6123	0.2523	1.1801	0.6371	0.2196	1.1385	0.6592	0.1900	1.0862	0.6787
0.4000	0.1136	0.3805	0.7266	0.0844	0.3285	0.7459	0.0618	0.2758	0.7605	0.0448	0.2261	0.7715	0.0321	0.1814	0.7796
0.6000	0.0414	0.1531	0.7735	0.0256	0.0971	0.7837	0.0155	0.0682	0.7901	0.0092	0.0464	0.7941	0.0054	0.0307	0.7965
0.8000	0.0153	0.0485	0.7902	0.0078	0.0295	0.7950	0.0039	0.0171	0.7975	0.0019	0.0096	0.7988	0.0009	0.0052	0.7994
1.0000	0.0057	0.0180	0.7964	0.0024	0.0090	0.7985	0.0010	0.0043	0.7994	0.0004	0.0020	0.7997	0.0002	0.0009	0.7999

Moody's development is based upon an initial configuration given by the relation $h = H_m [1 - (\frac{x_m}{L})^4]$. His origin of coordinates is at midspan. The following notation applies to his development.

$D_m = (d + H_m)$.

H_m represents the initial drainage depth, at midspan, at time zero.

h_m the drainable depth, at midspan, at the time t .

$m = \frac{H_m}{d + H_m} = \frac{H_m}{D_m}$.

q_m = the rate of discharge from the area between two drains, per unit length along the drains.

W_m = the total quantity of water removed from the area between two drains, per unit length along the drains, up to the time t .

x_m = distance from midspan measured horizontally toward a drain.

$\alpha_m = \frac{KD_m}{V}$

Table 8-1--Continued

 $m = 0$

$\frac{\alpha_m t}{L^2}$	$\frac{h_m}{H_m}$	$\frac{q_m L}{K D_m H_m}$	$\frac{W_m}{V L H_m}$
0.	1.	16.	0.
0.001	0.9998	12.9402	0.0139
0.002	0.9992	11.8719	0.0263
0.003	0.9983	11.1239	0.0377
0.004	0.9969	10.5388	0.0486
0.005	0.9952	10.0560	0.0589
0.006	0.9931	9.6447	0.0687
0.007	0.9906	9.2866	0.0782
0.008	0.9877	8.9698	0.0873
0.009	0.9845	8.6862	0.0961
0.010	0.9808	8.4298	0.1047
0.020	0.9279	6.7266	0.1795
0.030	0.8579	5.7562	0.2415
0.040	0.7844	5.0790	0.2955
0.050	0.7137	4.5459	0.3436
0.060	0.6478	4.0957	0.3867
0.070	0.5874	3.7014	0.4257
0.080	0.5324	3.3496	0.4609
0.090	0.4825	3.0332	0.4928
0.100	0.4372	2.7475	0.5217
0.200	0.1629	1.0238	0.6963
0.300	0.0607	0.3816	0.7613
0.400	0.0226	0.1422	0.7856
0.500	0.0084	0.0530	0.7946
0.600	0.0031	0.0198	0.7980
0.700	0.0012	0.0074	0.7993
0.800	0.0004	0.0027	0.7997
0.900	0.0002	0.0010	0.7999
1.000	0.0001	0.0004	0.8000

Parallel drains from the Laplace standpoint*

Consider the expression

$$p = (d-y) + \sum_{n=1}^{n=m} A_n \cosh \left(\frac{n\pi y}{L} \right) \sin \left(\frac{n\pi x}{L} \right) \quad (8-14)$$

where p represents a pressure. It is measured in feet of water and represents a departure from the pressures appropriate to a static state. This is a solution of the Laplace differential equation.

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0$$

which meets the requirement that $\frac{\partial p}{\partial y} = 0$ when $y = 0$. The coordinate y is measured upward from the barrier. The quantities A_n are to be chosen to meet the initial conditions that

$$p = H \quad \text{when } y = d \quad \text{for } 0 < x < L$$

The term $(d-y)$ represents the hydrostatic pressure which would be present if the water table were at the level of the water surface maintained in the drains. The quantity p represents a pressure in feet of water and the terms under the summation sign represent the additional pressures present when the water table is above the level of the drains. The pressure p will be zero when

$$0 = (d-y) + \sum_{n=1}^m A_n \cosh \left(\frac{n\pi y}{L} \right) \sin \left(\frac{n\pi x}{L} \right) \quad .$$

* This presentation follows closely that of the paper on "Parallel Drains from the Laplace Standpoint," by Robert E. Glover, which appeared in the Journal of the American Water Resources Association, Vol. 8, No. 1, February 1972, pp. 50-54 inclusive. The development is presented here through the courtesy of the A.W.R.A.

If the quantity n has an upper limit m , which implies a finite number of terms in the series, and $\frac{n\pi y}{L}$ is everywhere small compared to unity; then

$$\cosh\left(\frac{n\pi y}{L}\right) \cong 1$$

$$y_0 = d + \sum_{n=1}^m A_n \sin\left(\frac{n\pi x}{L}\right) .$$

This expression represents a water table profile. If a uniform increment of depth H reaches the water table, due to deep percolation from a uniform application of irrigation water, then the pressure imposed at drain level by the water table profile can be represented initially by the expression

$$p_0 = \frac{4H}{\pi} \sum_{n=1,3,5,\dots}^{n=\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right) . \quad (8-15)$$

If the series is terminated at the m th term then the expression can represent approximately a uniform increment of depth H . The relationships considered up to this point do not involve the element of time. This factor can now be introduced.

After the increment is applied, water will flow to the drains and the water table will begin to sink. The flows to each drain accounted for by the individual terms of the series will be:

$$\begin{aligned} K \int_0^d \left(\frac{\partial p}{\partial x}\right)_0 dy &= A_n K \frac{n\pi}{L} \int_0^d \cosh\left(\frac{n\pi y}{L}\right) dy = A_n K \sinh\left(\frac{n\pi y}{L}\right) \Big|_0^d \\ &= A_n K \sinh\left(\frac{n\pi d}{L}\right) \end{aligned}$$

The volume above the line $y = d$ is, approximately, for each term

$$S_n \cong \int_0^L (y-d) dx = A_n \cosh\left(\frac{n\pi d}{L}\right) \int_0^L \sin\left(\frac{n\pi x}{L}\right) dx \cong 2 A_n \frac{L}{n\pi} \cosh\left(\frac{n\pi d}{L}\right).$$

The continuity condition for each term is, since there is flow out at $x = 0$ and at $x = L$

$$V \frac{\partial S_n}{\partial t} = 2 K \int_0^d \left(\frac{\partial p}{\partial x}\right)_0 dy$$

In this expression t represents time. By substitution:

$$\frac{dA_n}{dt} \frac{2LV}{n\pi} \cosh\left(\frac{n\pi d}{L}\right) = 2 A_n K \sinh\left(\frac{n\pi d}{L}\right)$$

or if

$$\beta = \frac{Kn\pi}{LV} \tanh\left(\frac{n\pi d}{L}\right)$$

$$\frac{dA_n}{dt} + \beta A_n = 0$$

This is a differential equation whose solution is:

$$A_n = B_n e^{-\beta t}$$

Where the B_n quantities are new constants. Then all of the requirements described previously will be met to a close approximation if

$$p \cong \frac{4H}{\pi} \sum_{n=1,3,5,\dots}^{n=m} \frac{e^{-\beta t}}{n} \sin\left(\frac{n\pi x}{L}\right) \frac{\cosh\left(\frac{n\pi y}{L}\right)}{\cosh\left(\frac{n\pi d}{L}\right)} + (d-y) \quad (8-16)$$

This expression remains an exact solution of equation 1.

When

$$\frac{n\pi d}{L} \ll 1 \quad \tanh\left(\frac{n\pi d}{L}\right) \cong \frac{n\pi d}{L}$$

and with

$$\beta \cong \frac{\alpha n^2 \pi^2}{L^2} \quad \alpha = \frac{Kd}{V}$$

Then, approximately,

$$h \cong \frac{4H}{\pi} \sum_{n=1,3,5,\dots}^{n=m} \frac{e^{-n^2 \pi^2 \left(\frac{\alpha t}{L^2}\right)}}{n} \sin\left(\frac{n\pi x}{L}\right) + (d-y) .$$

The formula derived from the Dupuit-Forchheimer idealization is then recovered but with the important exception that here

$$\alpha = \frac{Kd}{V} \tag{8-17}$$

If the solution is limited to three terms and $\frac{\pi y}{L} < 0.1$ everywhere then the Dupuit-Forchheimer and Laplace solutions become essentially identical in form and an approximation to the initial condition is obtained which is close enough for practical purposes. Since the higher ordered terms vanish rapidly it is permissible to use Table 10 for computation of the remaining drainable depth at $x = L/2$ except that here the aquifer constant must be of the form $\alpha = Kd/V$.

Examples

Use of the first approximation solution and the Laplace type solution will be illustrated by use of an example adapted from Dumm's 1964 paper. The given data are:

Depth from ground surface to barrier	30 ft
Depth from ground surface to drain	8 ft

Drainable depth produced by an irrigation	0.46 ft
Permeability	$K = 10$ ft/day
Effective voids ratio	0.18
Drain spacing	$L = 1450$ ft

We will compute the remaining drainable depth at the point midway between drains at the end of successive seven day periods. Values for three months, six months, and one year are added. The differences between the values obtained by the first approximation and the Laplace type solution are due only to the difference in the α values. These differences are small. The drainable depth at the center does not respond immediately to the action of the drains. The part remaining, however, responds very quickly due to drainage taking place in the immediate neighborhood of the drain.

Computation by first approximation

$$K = 10 \text{ ft/day} \quad D_a = \left(d + \frac{H}{2}\right) = \left(22 + \frac{0.46}{2}\right) = 22.23 \text{ ft}$$

$$V = 0.18 \quad \alpha = \frac{KD_a}{V} = \frac{(10)(22.23)}{0.18} = 1235 \frac{\text{ft}^2}{\text{day}} \quad L = 1450 \text{ ft}$$

$$\frac{\alpha}{L^2} = \frac{1235}{1450^2} = 0.0005874 \frac{1}{\text{day}} \quad H = 0.46 \text{ ft}$$

Time Days	$\frac{\alpha t}{L^2}$	$\frac{h_c}{H}$	h_c	p	Remarks
0	0	1.000	0.460	1.000	Values of $\frac{h_c}{H}$ can be
7	0.004112	1.000	0.460	0.855	read from Table 10.
14	0.008224	1.000	0.460	0.795	Values of p can be
21	0.012335	0.997	0.459	0.749	read from Table 11.
28	0.016447	0.988	0.454	0.711	
35	0.020559	0.973	0.448	0.676	
42	0.024671	0.951	0.473	0.646	
49	0.028782	0.926	0.426	0.617	
56	0.032894	0.897	0.413	0.591	
63	0.037006	0.868	0.399	0.566	

Time Days	$\frac{\alpha t}{L^2}$	$\frac{h_c}{H}$	h_c	p	Remarks
70	0.041118	0.838	0.385	0.543	
77	0.045229	0.807	0.371	0.520	
84	0.049341	0.777	0.357	0.479	
91	0.053453	0.748	0.344	0.499	Three months
182	0.106906	0.443	0.204	0.282	Six months
365	0.214400	0.153	0.070	0.098	One year

Computation by the Laplace type solution

$$\alpha = \frac{Kd}{V} = \frac{(10)(22)}{0.18} = 1222 \text{ (ft}^2\text{/day)} \quad (\alpha/L^2) = 0.0005813$$

Time Days	$\frac{\alpha t}{L^2}$	$\frac{h_c}{H}$	h_c	p	Remarks
0	0	1.000	0.460	1.000	
7	0.004069	1.000	0.460	0.845	
14	0.008138	1.000	0.460	0.796	
21	0.012207	0.997	0.459	0.751	
28	0.016276	0.989	0.455	0.712	
35	0.020346	0.974	0.448	0.678	
42	0.024417	0.953	0.438	0.647	
49	0.028484	0.928	0.427	0.619	
56	0.032553	0.900	0.414	0.593	
63	0.036622	0.871	0.401	0.568	
70	0.040691	0.841	0.387	0.545	
77	0.044760	0.811	0.373	0.523	
84	0.048829	0.781	0.359	0.502	
91	0.052898	0.752	0.346	0.482	Three months
182	0.105797	0.448	0.206	0.285	Six months
365	0.212174	0.157	0.072	0.100	One year

$$\text{Note: } \frac{\pi y_m}{L} = \frac{(3.1416)(22.46)}{1450} = 0.0487$$

These two examples yield closely similar results because the drainable depth is small compared to the saturated depth below the drains. It will be

profitable to now consider a somewhat extreme case where the drainable depth is nearly equal to the saturated depth below the drains and to again compare the results of computations made by several methods.

As an example of a case where the drains are located about midway between the water table and the barrier, data from a field installation supplied by Mr. Ray Winger of the Bureau of Reclamation will be used. The data are:

Depth of barrier below ground surface	16 ft
Drain depth	9 ft
Permeability	1.4 (ft/day)
Effective voids ratio	0.093
Maximum allowable water table height 3 ft below ground surface or 6 ft above the drains	
Drain spacing	510 ft

Computation of the drainable depth midway between drains by the method of Moody.

$$D_m = (16 - 3) = 13 \text{ ft.} \quad \alpha_m = \frac{(1.4)(13)}{0.093} = 195.7 (\text{ft}^2/\text{day}).$$

$$m = \frac{6}{13} = 0.462.$$

Time Days .	$\frac{\alpha_m t}{L^2}$	$\frac{h_m}{H_m}$	h_m
0	0	1.000	6.00
20	.0150	.960	5.76
40	.0301	.882	5.29
60	.0451	.809	4.85
80	.0602	.736	4.42
100	.0752	.666	4.00
120	.0903	.606	3.64
140	.1053	.556	3.34
160	.1204	.506	3.04
180	.1354	.463	2.78
270	.2031	.328	1.97
365	.2746	.227	1.36

Time	$\frac{\alpha_m t}{L^2}$	$\left(\frac{h_m}{H_m}\right)$	h_m
42	0.7584	0.015	0.09
49	0.8848	0.008	0.05

One day is 24 hours. $\frac{\alpha_m t_1}{L^2} = \frac{(195.7)(24)}{510^2} = \frac{4696.8}{260100} = 0.018057$

Computation by first approximation method.

$$D_a = \left(d + \frac{H_o}{2}\right) = \left(7 + \frac{6}{2}\right) = 10 \text{ ft} \quad \alpha = \frac{KD_a}{V} = \frac{(1.4)(10)}{0.093} = 150.5 \text{ (ft}^2/\text{day)}.$$

$$\frac{\alpha t_1}{L^2} = \frac{(150.54)(24)}{510^2} = 0.01389$$

Time Days	$\left(\frac{\alpha t}{L^2}\right)$	$\left(\frac{h_c}{H_o}\right)^*$	h_c
0	0	1.0000	6.00
20	.0116	.9979	5.99
40	.0231	.9599	5.76
60	.0347	.8844	5.31
80	.0463	.7992	4.80
100	.0579	.7164	4.30
120	.0694	.6409	3.85
140	.0816	.5687	3.41
160	.0926	.5103	3.06
180	.1042	.4551	2.73
270	.1562	.2724	1.63
365	.2112	.1583	0.95

* Read from Table 10.

Computation by the method of Brooks.

Time Days	h_c	$(h_c - \frac{H_o}{2})$	h_1	$(h_1 + \frac{H_o}{2})$
0	6.00	3.00	3.00	6.00
20	5.99	2.99	2.99	5.99
40	5.76	2.76	2.81	5.81
60	5.31	2.31	2.46	5.46
80	4.80	1.80	2.04	5.04
100	4.30	1.30	1.62	4.62
120	3.85	0.85	1.22	4.22
140	3.41	0.41	0.83	3.83
160	3.06	0.06	0.50	3.50
180	2.73	-0.27	0.18	3.18
270	1.63	-1.37	-0.96	2.04
365	0.95	-2.05	-1.75	1.25

Notes: The numbers in the column headed h_c are those of the first approximation. The numbers in the column headed $(h_c - \frac{H_o}{2})$ are those of the first approximation referred to an origin $(H_o/2)$ above the level of the drains. The column headed h_1 is Brooks second approximation, computed by use of formula 8-10. The figures in the last column are those of the previous column referred back to drain level. They compare with the first approximation figures in the column headed h_c .

Reference: Brooks, R.H., 1963, ASCE Paper 3420.

Laplace type solution.

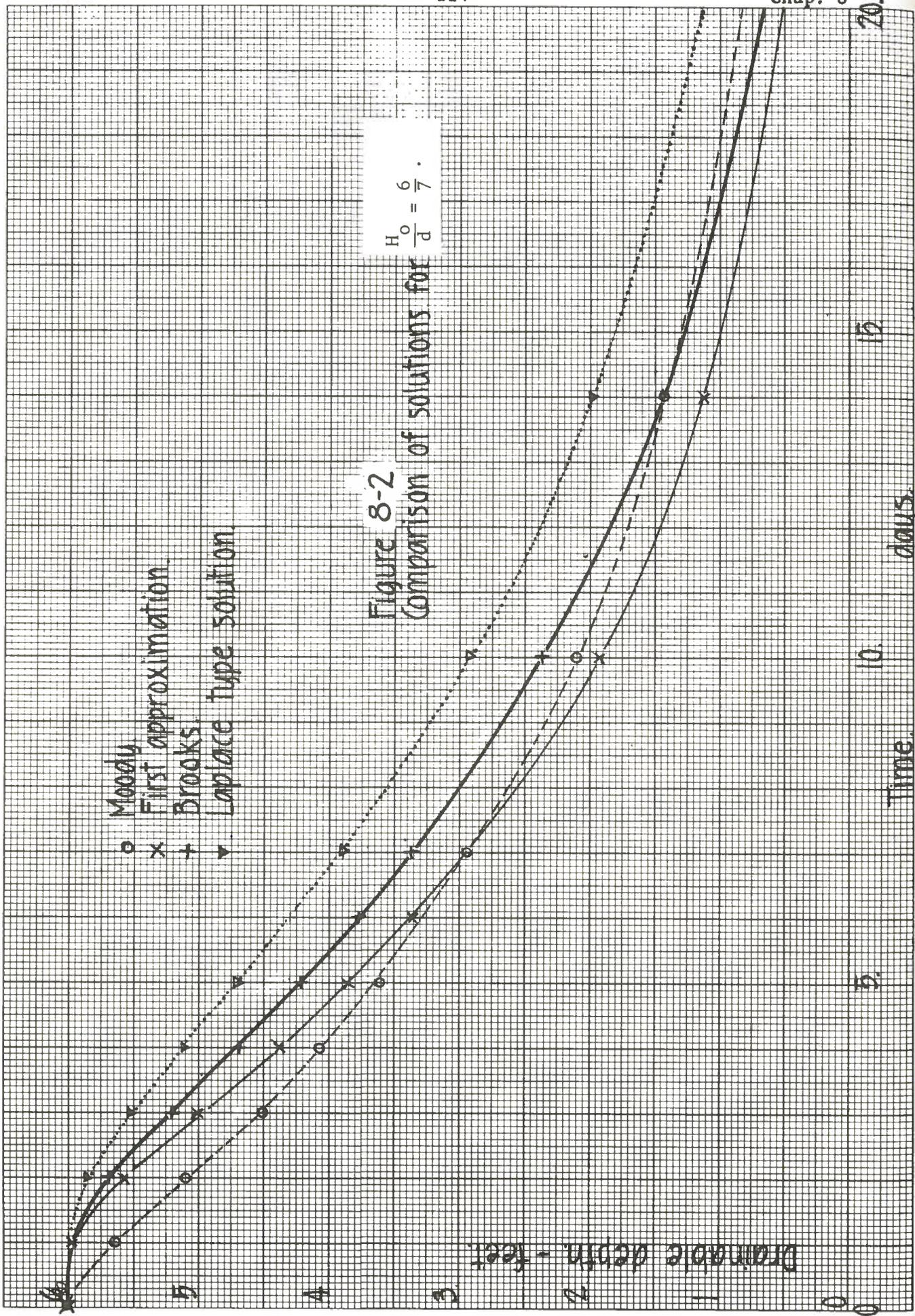
$$d = 7.0 \text{ ft} \quad \alpha_L = \frac{Kd}{V} = \frac{(1.4)(7.0)}{0.093} = 105.4 \text{ (ft}^2\text{/day)}$$

$$L = 510 \text{ ft} \quad H_o = 6.00 \text{ ft}$$

Time Days	$\frac{\alpha_L t}{L^2}$	$\frac{h_c}{H_o}$	h_c
0	0	1.000	6.00
20	.0081	1.000	6.00
40	.0162	.989	5.93
60	.1243	.953	5.72
80	.0324	.901	5.40
100	.0405	.842	5.05
120	.0486	.782	4.69
140	.0567	.725	4.35
160	.0648	.670	4.02
180	.0729	.619	3.72
270	.1094	.432	2.60
365	.1479	.296	1.77

Note: The values in the column headed (h_c/H_o) were obtained from Table 10.

The results of these computations are shown on figure 8-2. The solid heavy line represents Brooks second approximation which will here be used as a basis for comparison. The light solid line shows the results obtained by use of the first approximation solution. It holds up surprisingly well even though here the drainable depth is almost half of the original saturated depth and nearly equal to the saturated depth below the drains. The dashed curve shows the results obtained from using Moody's computer solution. This solution and the first and second approximation solutions are not strictly comparable because they have different initial conditions. The initial



condition of Moody's solution, however, represents closely a configuration which would appear at an early epoch in the drainage of a uniform drainable depth. If Moody's initial configuration is superimposed on such a chart as that of figure 7 of USBR Monograph 31, it will be found to correspond nearly to the profile for $\frac{\alpha t}{L^2} = 0.014$. A second approximation curve obtained by use of Brooks formula indicates that the parameter should be about $\frac{\alpha t}{L^2} = 0.010$ to produce a close fit. Since the value of $\frac{\alpha t_1}{L^2} = 0.015$ for time 20 days is substantially this amount it can be concluded that if Moody's curve is shifted to the right about 20 days on figure 8-2 the effect of the differing initial conditions will be accounted for. If this is done Moody's result and Brook's second approximation will be in close agreement over the first ten days. The solution obtained from the Laplace formulation is similar in shape to the second approximation curve of Brooks but lies above it. The reason for this seems to be that this solution accounts for the head loss needed to produce vertical as well as horizontal flow whereas the other solutions account for the horizontal component of flow only. A particle of water initially at the water table ten feet back from the drain, for example, has to travel six feet vertically and 10 feet horizontally to reach the drain. The solution of the Laplace equation accounts for this but the solutions derived on the Dupuit-Forchheimer basis only account for the horizontal ten feet of distance. The drainage is therefore slowed near the drain and the drainage of water remote from the drain is also slowed because it cannot reach the drain until the water close to the drain is disposed of. This comparison brings out the important effect of flow convergence near the drain. More will be said on this point later.

Selection of drain spacings

The formulas described can be used as a means for computing drain spacings on a cut and try basis. The procedure will be illustrated by use of the first approximation formula. The h_c/H values will be obtained from Table 10. The computation will be based upon figures adapted from Lee D. Dumm's 1964 paper. The allowable rise of the water table at mid-span at the end of the irrigation season is 4 ft. It will be assumed that this height is attained at the end of the previous irrigation season.

Data are:

$$K = 10 \text{ ft/day}$$

$$D = 22 \text{ ft}$$

$$V = 0.18$$

$$\frac{\alpha}{L^2} = \frac{1222.23}{1500^2} = 0.00054321$$

$$KD = 220 \text{ ft}^2/\text{day}$$

$$\alpha = \frac{KD}{V} = 1222.23 \text{ ft}^2/\text{day}$$

Try a spacing of 1500 feet.

Application	Time Days	Drainable Depth ft	$\left(\frac{\alpha t}{L^2}\right)$	$\left(\frac{h_c}{H}\right)$	h_c
Apr 22*	132	0.46	0.0717	0.6267	0.288
June 6	87	0.46	0.0473	0.7920	0.364
July 1	62	0.46	0.0337	0.8887	0.409
July 21	42	0.46	0.0228	0.9616	0.442
Aug 4	28	0.46	0.0152	0.9917	0.456
Aug 18	14	0.46	0.0076	0.9998	0.460
Sept 1	0	0.46	0	1.0000	0.460
	365	4.00	0.1983	0.1799	<u>0.720</u>
			Total		3.599

*Snowmelt

This spacing can be widened. Try a spacing of 1700 ft.

Application	Time Days	Drainable Depth ft	$\left(\frac{\alpha t}{L^2}\right)$	$\left(\frac{h_c}{H}\right)$	h_c
Apr 22*	132	0.46	0.0558	0.731	0.336
June 6	87	0.46	0.0368	0.869	0.400
July 1	62	0.46	0.0262	0.942	0.433
July 21	42	0.46	0.0178	0.984	0.453
Aug 4	28	0.46	0.0118	0.998	0.459
Aug 18	14	0.46	0.0059	1.000	0.460
Sept 1	0	0.46	0	1.000	0.460
	365	4.00	0.1544	0.277	<u>1.110</u>
			Total		4.111

*Snowmelt

This spacing is too wide. By interpolation, a spacing of 1657 feet would just meet the requirements of a four-foot rise at the end of the irrigation season.

The method of M. Maasland

The previous treatments of drainage by parallel drains has been based upon the concept of a drainable depth which comes into existence at time zero. This is an idealization of furrow irrigation practices where the irrigation water is applied during a brief interval of time and produces, by deep percolation, a drainable depth H .

The Maasland approach is somewhat different and possesses certain advantages which will be described later. He assimilates the deep percolation from a succession of irrigations to an average infiltration rate i . The consequences of such an idealization may be approached through the use of the formula for drainage of a single uniform drainable depth H (f. 8-1). This is:

$$h = \frac{4H}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n} \sin\left(\frac{n\pi x}{L}\right)$$

Suppose the drainable depth dH appears at the time ξ where ξ represents a time variable running between 0 and t . The variable ξ indicates the time of occurrence of an event whose effect is to be computed at the time t .

The drainable depth at the point x at the time t will then be given by

$$h = \frac{i}{V} \int_0^t \frac{4}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha(t-\xi)}{L^2}\right)}}{n} \sin\left(\frac{n\pi x}{L}\right) d\xi$$

Where $\frac{i}{V}$ is the rate of rise of the water table due to the constant infiltration rate i . The infiltration is considered to be entire water.

By integration

$$h = \frac{i4}{V\pi} \frac{L^2}{\alpha\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha(t-\xi)}{L^2}\right)}}{n^3} \sin\left(\frac{n\pi x}{L}\right) \Bigg|_0^t$$

or

$$h = \frac{i4L^2}{Kd\pi^3} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^3} \sin\left(\frac{n\pi x}{L}\right) - \frac{i4L^2}{Kd\pi^3} \sum_{n=1,3,5,\dots}^{\infty} \frac{e^{-n^2\pi^2\left(\frac{\alpha t}{L^2}\right)}}{n^3} \sin\left(\frac{n\pi x}{L}\right) \quad (8-18)$$

At $x = L/2$, the point midway between the drains,

$$\frac{n\pi x}{L} = \frac{n\pi}{2}$$

Also

$$\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots = \frac{\pi^3}{32}$$

This is one of the Euler numbers. A plot of formula 8-18 is shown on figure 8-3.

The summation of descending exponentials disappears in time. Then there is an ultimate steady state given by the summation which is free of exponentials. This is:

$$h_c = \frac{i4L^2}{Kd\pi^3} \sum_{n=1,3,5,\dots}^{n=\infty} \frac{1}{n^3} \sin\left(\frac{n\pi}{2}\right) = \frac{i4L^2}{Kd\pi^3} \frac{\pi^3}{32} = \frac{iL^2}{8Kd} \quad (8-19)$$

An independent development for the steady state will be of interest.

The statement that the flow is equal to the supply is

$$Kd \frac{dh}{dx} = i \left(\frac{L}{2} - x\right).$$

By integration, if $h = 0$ when $x = 0$

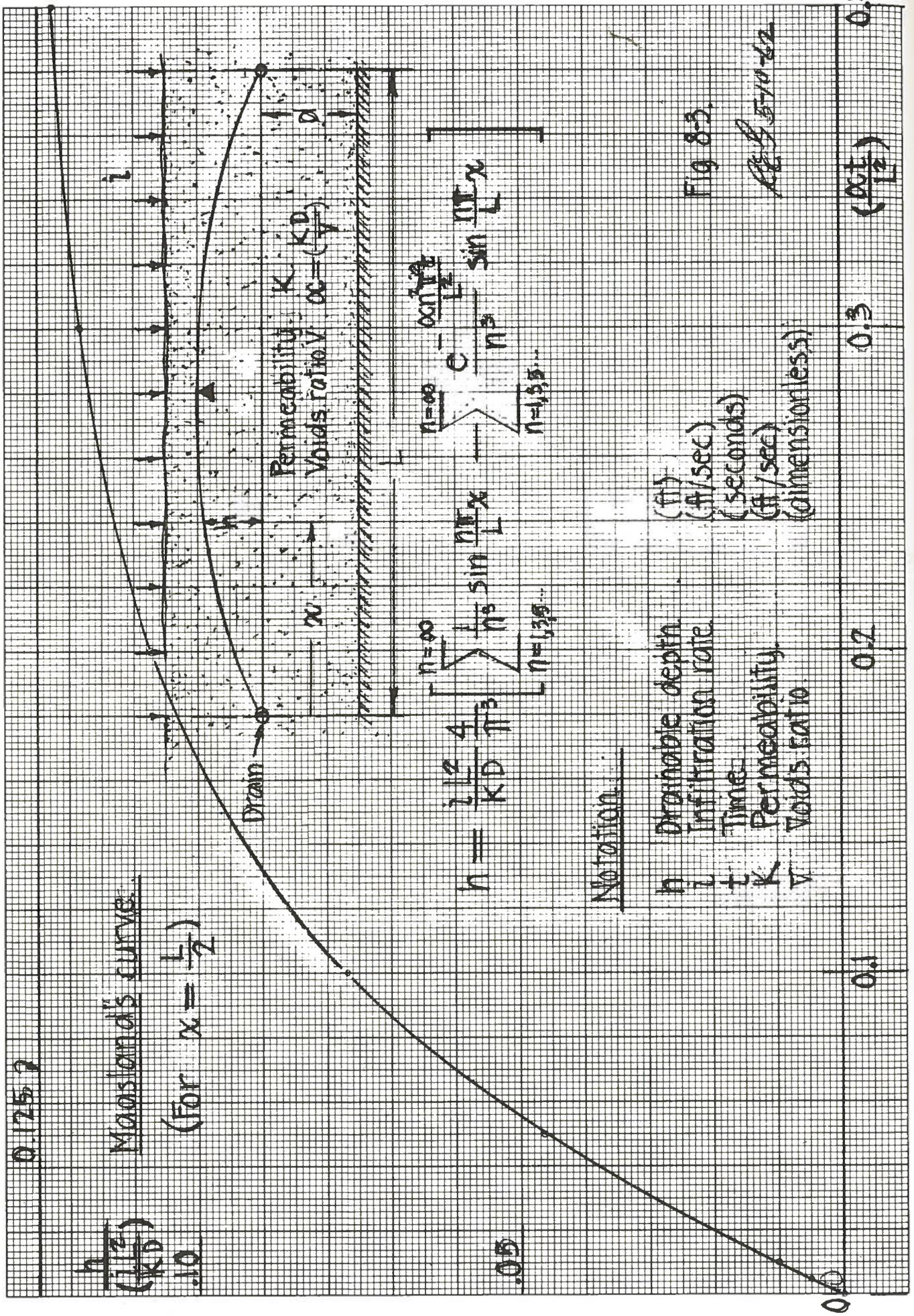
$$h = \frac{i}{2Kd} x(L - x).$$

When $x = \frac{L}{2}$

$$h_c = \frac{iL^2}{8Kd} \quad (8-20)$$

as before. It will be found that the term

$$\frac{i4L^2}{V\alpha\pi^3} \sum_{n=1,3,5,\dots}^{n=\infty} \frac{1}{n^3} \sin\left(\frac{n\pi x}{L}\right) \quad \text{is the Fourier series}$$



which represents

$$h = \frac{i}{2Kd} \times (L - x) .$$

Formulas of the type of equation 8-20 have been used to estimate drain spacings. It will be clear that this procedure implies that the irrigation season is long enough to establish the ultimate steady state. Under ordinary conditions the irrigation season is too short to establish an ultimate steady state and the result is that drain spacings obtained by use of ultimate steady state relations are closer than necessary to provide drainage. To put this in other words, the use of ultimate steady state formulas, based upon the concept of a continuous infiltration rate i , neglects the favorable effects of the winter drain-out period.

This difficulty can be substantially overcome if the effects of a succession of seasonal applications are considered. This is a case of intermittent operation as treated in Chapter 11. With an irrigation pattern as illustrated on figure 8-4 the effects of previous irrigations and cessation of irrigations can be treated in the following way. The height of the water table midway between drains at the end of the last irrigation season is of interest. Here T represents the yearly period and $T/3$ the irrigation period.

Suppose

$$(\alpha T/L^2) = 0.2$$

then the computation is made in the following way.

Fig. 8.4 Drainable depth at center of drain spacing.

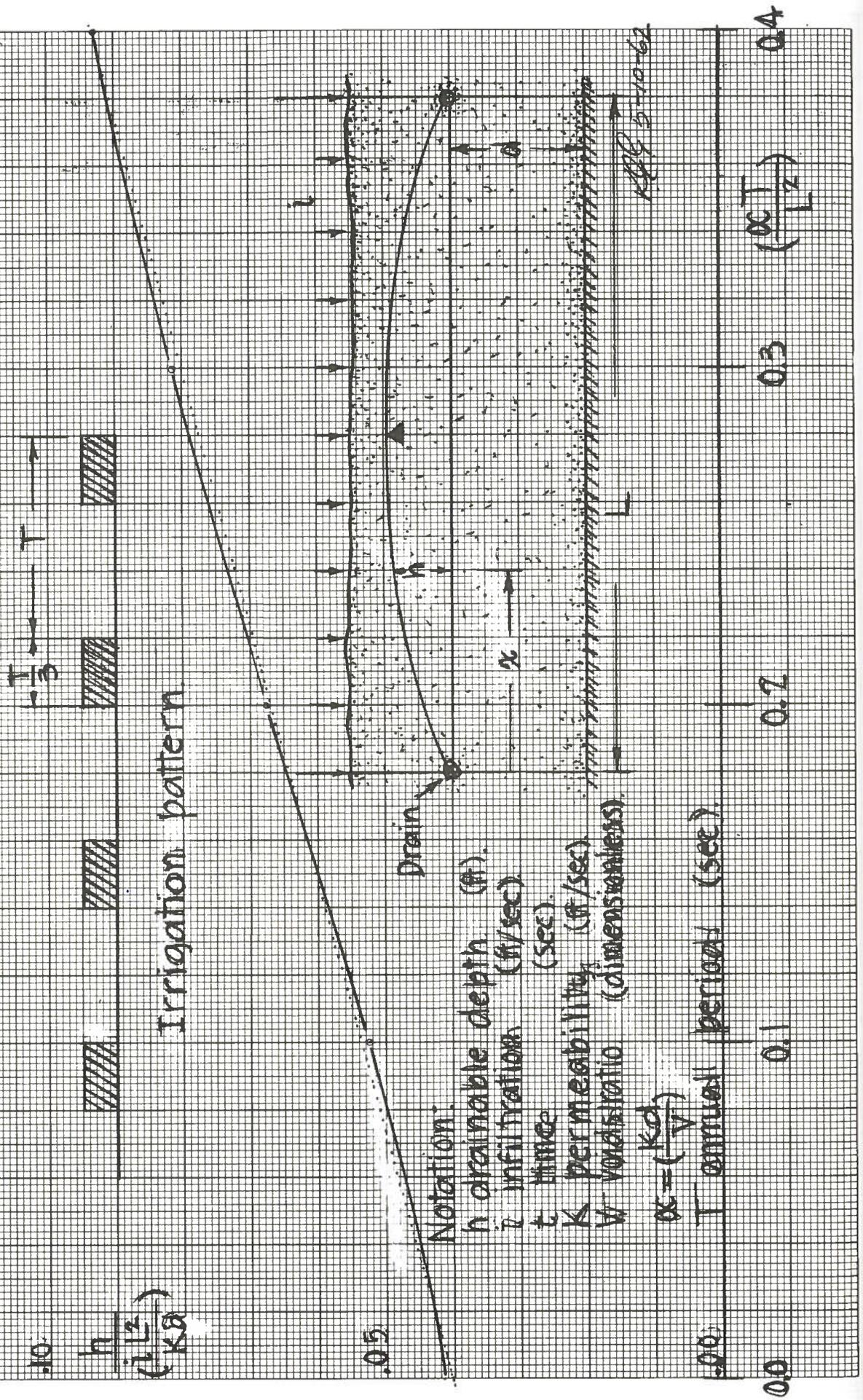


Table 8-2 Computation of $h/\left(\frac{iL^2}{Kd}\right)$ for intermittent irrigation. $(\alpha T/L^2) = 0.2$.

Irrigation period $T/3$.

$(\alpha t/L^2)$	$h/\left(\frac{iL^2}{Kd}\right)$
0.0667	+0.0585
0.2000	-0.1071
0.2667	+0.1137
0.4000	-0.1225
0.4667	+0.1250
Total	+0.0676

(Compare with figure 8-4.)

The first figure represents the effect at the end of the last irrigation period which is 1/3 of a year in length. Then

$$(\alpha t/L^2) = (0.2/3) = 0.0667$$

The corresponding value for $h/\left(\frac{iL^2}{Kd}\right)$ is obtained from figure 8-3. The next figure represents a cessation of irrigation at the end of the previous irrigation period. Here $(\alpha t/L^2) = 0.2000$ because the time is one year. The third figure with $(\alpha t/L^2) = (4)(0.2)/3 = 0.2667$ accounts for the beginning of the previous irrigation period. The fourth and fifth figures, together, account for the irrigations made two years previous. As the $(\alpha t/L^2)$ values grow larger the two values of the pair approach equality. This explains how convergence can be obtained even though the values are growing larger with time. A series of such computations will permit the construction of a chart such as shown on figure 8-4. With specified values of i , L , K , d , α , T an $(\alpha T/L^2)$ value can be computed and an $h/\left(\frac{iL^2}{Kd}\right)$ value can be read directly from the chart which includes the effects of irrigations in previous years. An h value can then immediately be computed. A cut and try procedure for

estimating drain spacings can then be used which will take account of the effects of irrigations in previous years.

A direct approach to this problem can be made if it is noted that the graph of figure 8-4 is nearly a straight line. The straight line approximation shown has the formula

$$\frac{h_c}{\left(\frac{iL^2}{Kd}\right)} = 0.040 + 0.1325 \left(\frac{\alpha T}{L^2}\right) \quad \text{For } 0 < \left(\frac{\alpha T}{L^2}\right) < 0.4$$

From which, by rearrangement,

$$\left(\frac{L^2}{\alpha T}\right) = 25 \left(\frac{h_c V}{i T}\right) - 3.3125 \quad (8-21)$$

The problem of determining a drain spacing for the conditions of the problem used to illustrate application of the first approximation solution of Chapter 8 may now be reconsidered. With

$$\begin{aligned} K &= 10 \text{ ft/day} & V &= 0.18 \\ D_a &= 22.225 \text{ ft} & K D_a &= 222.25 \text{ ft/day} \\ \alpha &= 1234.7 \text{ ft}^2/\text{day} \end{aligned}$$

Irrigation applications contributing 0.46 ft of drainable depth were made on June 6, July 1, July 21, August 4, August 18 and September 1. A similar contribution from snowmelt was indicated for April 22. In all (7) (0.46) = 3.22 feet of drainable depth were contributed in 132 days. To accommodate this to our chart conditions we can assume that these applications were made in 1/3 year or 122 days. Then

$$i = \frac{(3.22)(0.18)}{122} = 0.00475 \frac{\text{ft}}{\text{day}}$$

The allowable drainable depth at the center of the span is 4.0 ft and $T = 365$ days. Then

$$\left(\frac{hV}{iT}\right) = \frac{(4.0)(0.18)}{(0.00475)(365)} = \frac{0.72}{1.734} = 0.4152$$

$$\left(\frac{L^2}{\alpha T}\right) = (25)(0.4152) - 3.3125 = 7.0675$$

$$L^2 = (7.0675)(1234.7)(365) = 3185100$$

$$L = 1785 \text{ feet}$$

This compares with Dumm's estimate of 1450 feet. The difference is largely due to a difference in assumed intervals between applications. In our case it was about 17 days whereas his last three irrigations were made at 14 day intervals. A corresponding infiltration rate would be

$$i = \frac{(3)(0.46)(0.18)}{(3)(14)} = 0.00591 \frac{\text{ft}}{\text{day}}$$

and

$$\frac{hV}{iT} = \frac{(4.0)(0.18)}{(0.00591)(365)} = \frac{0.720}{2.157} = 0.3338$$

$$\frac{L^2}{\alpha T} = (25)(0.3338) - 3.3125 = 5.0325$$

$$L^2 = (5.0325)(1234.7)(365) = 2265000 \text{ ft}^2$$

$$L = 1506 \text{ feet}$$

The ultimate steady state formula would give

$$L = \sqrt{\frac{8 h_c K_d}{i}} = \sqrt{\frac{(8)(4.0)(222.25)}{0.00591}} = 1097 \text{ feet}$$

This is admittedly too short for the reasons mentioned previously. The trial procedure described in the paragraph on "Selection of drain spacings" is much shortened if the trial value is close. Chart 8-4 and formula 8-21 can provide good trial values.

Flow of water to drains

The flow of drainage water from the width L between drains to the two drains bordering the width is:

$$q_2 = 2Kd \left(\frac{\partial h}{\partial x} \right)_0 = \frac{8iL}{\pi^2} \sum_{n=1,3,5,\dots}^{n=\infty} \frac{1}{n^2} - \frac{8iL}{\pi^2} \sum_{n=1,3,5,\dots}^{n=\infty} \frac{e^{-n^2\pi^2 \left(\frac{\alpha t}{L^2} \right)}}{n^2}$$

Since the cosine terms which arise as a result of the indicated differentiations are 1 when $x = 0$. But

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

This is one of the Bernoulli numbers. Then

$$\frac{q_2}{iL} = 1 - \frac{8}{\pi^2} \sum_{n=1,3,5,\dots}^{n=\infty} \frac{e^{-n^2\pi^2 \left(\frac{\alpha t}{L^2} \right)}}{n^2} \quad (8-22)$$

This can be put in the form:

$$\frac{q_2}{iL} = 1 - p \quad (8-23)$$

where values of p can be obtained from Table 11.

As an example of the use of this result we may compute the flow of drainage water from the width between drains using the data of an example from Chapter 8. It is worthwhile to note that the flow so obtained will be appropriate for the flow of drainage water to one drain from both sides.

With

$$\alpha = 1222.23 \text{ ft}^2/\text{day} \quad \left(\frac{\alpha t}{L^2} \right) = 0.06627$$

$$L = 1500 \text{ ft}$$

$$i = 0.00591 \text{ ft/day}$$

$$t = 122 \text{ days}$$

From tables

$$p = 0.4217$$

$$1 - p = 0.5783$$

Then

$$q_2 = iL(1-p) = (0.00591) (1500) (0.5783) = 5.1266 \text{ ft}^2/\text{day}$$

This means that each drain must be able to pick up and carry away a little over five cubic feet of drainage water per foot of drain per day. This estimate can be expected to be below that obtained by the methods which account for the initial rush of water to the drains immediately following the application of irrigation water. Drains designed in this way could be expected to run at maximum capacity for a few days following irrigation. Some comparisons will be found in the paragraph on "Local convergence losses."

Local convergence losses

Where tile drains are used, the flow, which has been occupying the entire saturated depth, must converge toward the drain. This means that the flow must pass through restricted areas and it must be expected that increased head losses will be required to move the flow to the drain.

The following development has for its purpose the evaluation of these convergence losses.

Consider the expression (Byerly)

$$p_1 = \frac{p_0}{\pi} \log_e \left[\cosh^2 \left(\frac{\pi x}{d} \right) - \cos^2 \left(\frac{\pi y}{d} \right) \right]$$

where p_1 represents the pressure needed to drive the flow. It is measured in feet of water. It is a solution of the Laplace equation

$$\frac{\partial^2 p_1}{\partial x^2} + \frac{\partial^2 p_1}{\partial y^2} = 0$$

By differentiation

$$\frac{\partial p_1}{\partial x} = \frac{p_0}{\pi} \left[\frac{2 \cosh\left(\frac{\pi x}{d}\right) \sinh\left(\frac{\pi x}{d}\right)}{\cosh^2\left(\frac{\pi x}{d}\right) - \cos^2\left(\frac{\pi y}{d}\right)} \right] \frac{\pi}{d}$$

Then

$$\frac{\partial p_1}{\partial x} = 0 \quad \text{if } x = 0 \quad \text{when } y > 0$$

By differentiation with respect to y

$$\frac{\partial p_1}{\partial y} = \frac{p_0}{\pi} \left[\frac{2 \cos\left(\frac{\pi y}{d}\right) \sin\left(\frac{\pi y}{d}\right)}{\cosh^2\left(\frac{\pi x}{d}\right) - \cos^2\left(\frac{\pi y}{d}\right)} \right] \frac{\pi}{d}$$

Then

$$\begin{aligned} \frac{\partial p_1}{\partial y} &= 0 \quad \text{when } y = d \\ &= 0 \quad \text{when } y = 0 \quad \text{if } x > 0 \end{aligned}$$

The idealization is as shown in figure 8-5.

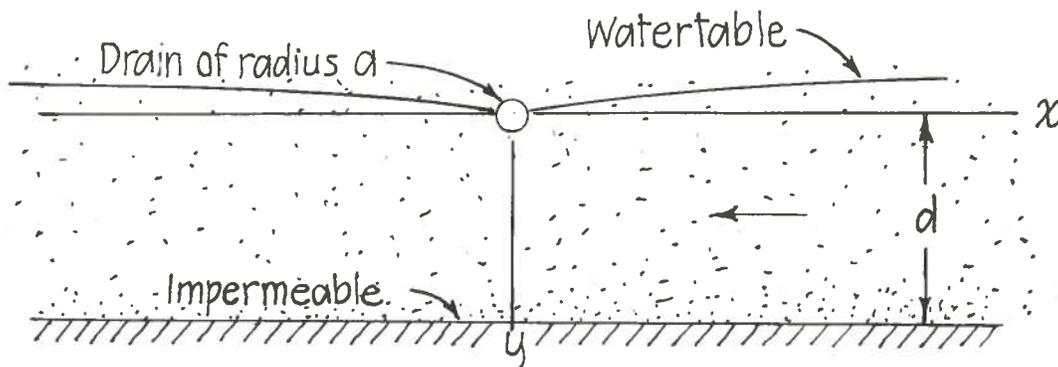


Fig. 8-5 Drain.

The origin is at the center of the drain and a sink is located there. The flow approaches from the right and goes to the sink without crossing the y axis. The idealization neglects the presence of saturated depth above the elevation of the drains. The tacit assumption is also made that the flow enters the drain through a quadrant. The quantity p_1 can be considered as the pressure which drives the flow. The flow approaches the drain along a strip of uniform width d . The pressure difference between the point $y = 0$ and x and the point $y = 0$ and $x = a$ is

$$p_1 - p_3 = \frac{p_0}{\pi} \log_e \frac{[\cosh^2 (\frac{\pi x}{d}) - 1]}{[\cosh^2 (\frac{\pi x}{a}) - 1]}$$

The pressure gain out to x due to the uniform flow in the strip is:

$$p_2 = \frac{2p_0 x}{d}$$

The gradient, when x is large compared to d , can be inferred from the expression for $(\partial p_1 / \partial x)$. When $(\pi x / d) \gg 1$ then $\cosh (\frac{\pi x}{d})$ and $\sinh (\frac{\pi x}{d})$ become large compared to unity and nearly equal while $\cos^2 (\frac{\pi y}{d})$ can never exceed unity. Then when $(\pi x / d) \gg 1$, $\frac{\partial p}{\partial x} \rightarrow 2 p_0 / d$. The above expression for p_2 can be derived from this result. It represents the head loss which would be needed to drive the flow

$$Kd \frac{\partial p_1}{\partial x} = 2Kp_0$$

from the point x to the origin if there were no convergence.

The pressure loss due to convergence is:

$$[p_1 - p_2 - p_3] = \frac{p_0}{\pi} \log_e [\cosh^2 (\frac{\pi x}{d}) - 1] - \frac{p_0}{\pi} \log_e [\cosh^2 (\frac{\pi a}{d}) - 1] - \frac{2x}{d}$$

when $(\frac{\pi x}{d})$ is large compared to unity then $\cosh^2(\frac{\pi x}{d})$ will be large compared to unity and

$$\cosh^2(\frac{\pi x}{d}) - 1 \cong \cosh^2(\frac{\pi x}{d}) \quad (\text{If } \frac{x}{d} \gg 1)$$

under these conditions also

$$\cosh^2(\frac{\pi x}{d}) \cong \frac{e^{(\frac{2\pi x}{d})}}{4} \quad (\text{If } \frac{x}{d} \gg 1)$$

then approximately

$$\frac{p_0}{\pi} \log_e [\cosh^2(\frac{\pi x}{d}) - 1] \cong \frac{p_0}{\pi} (\frac{2\pi x}{d}) \log_e e - \frac{p_0}{\pi} \log_e 4 = [\frac{2x}{d} - 0.44127] p_0$$

If (x/d) is large compared to unity the quantity 0.44127 can be dropped. If $(\pi a/d)$ is small compared to unity

$$\cosh^2(\frac{\pi a}{d}) \cong 1 + (\frac{\pi a}{d})^2 + \frac{1}{3} (\frac{\pi a}{d})^4 + \dots$$

and approximately

$$[\cosh^2(\frac{\pi a}{d}) - 1] \cong (\frac{\pi a}{d})^2$$

so that

$$\frac{p_0}{\pi} \log_e [\cosh^2(\frac{\pi a}{d}) - 1] \cong \frac{2p_0}{\pi} \log_e (\frac{\pi a}{d}) .$$

Finally

$$[p_1 - p_2 - p_3] \cong \frac{2p_0 x}{d} - \frac{2p_0}{\pi} \log_e (\frac{\pi a}{d}) - \frac{2p_0 x}{d}$$

Figures 8-6 and 8-7 are reproduced here through the courtesy of the American Concrete Institute.

These figures illustrate the close relationship of the mathematical treatments of the flow of heat in solids and the flow of groundwater. These charts first appeared in a paper on "Insulation for Protection of New Concrete in Winter" by L. H. Tuthill, R. E. Glover, C. H. Spencer and W. B. Bierce in the Journal of the Concrete Institute for November 1951. The charts appear on pages 262 and 264.

The local resistance incident to converging flow to a drain here replaces the resistance to heat flow produced by form insulation.

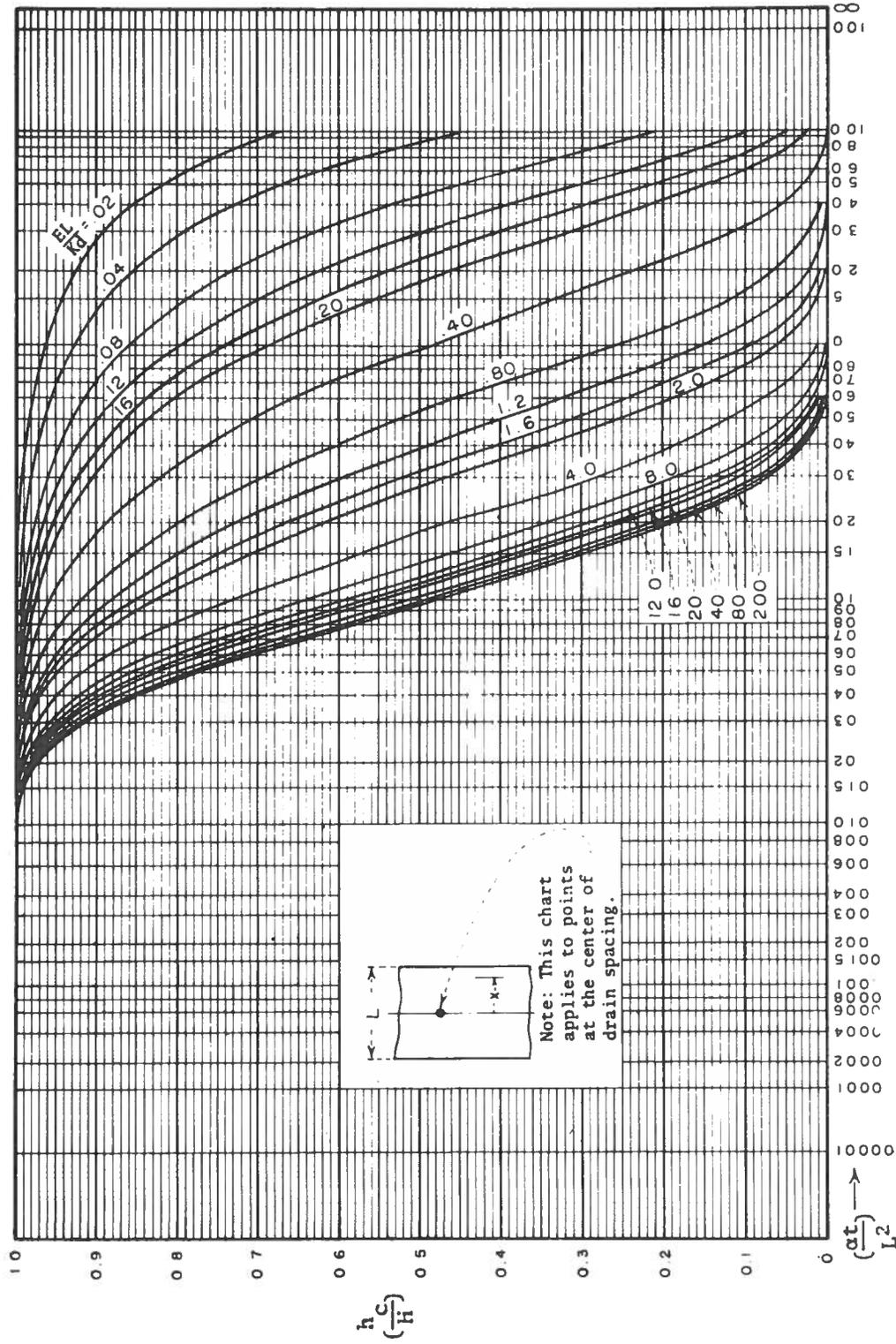


Fig. 8-6 (h_c/H) vs ($\alpha t/L^2$) taking account of local flow resistance at the drain.

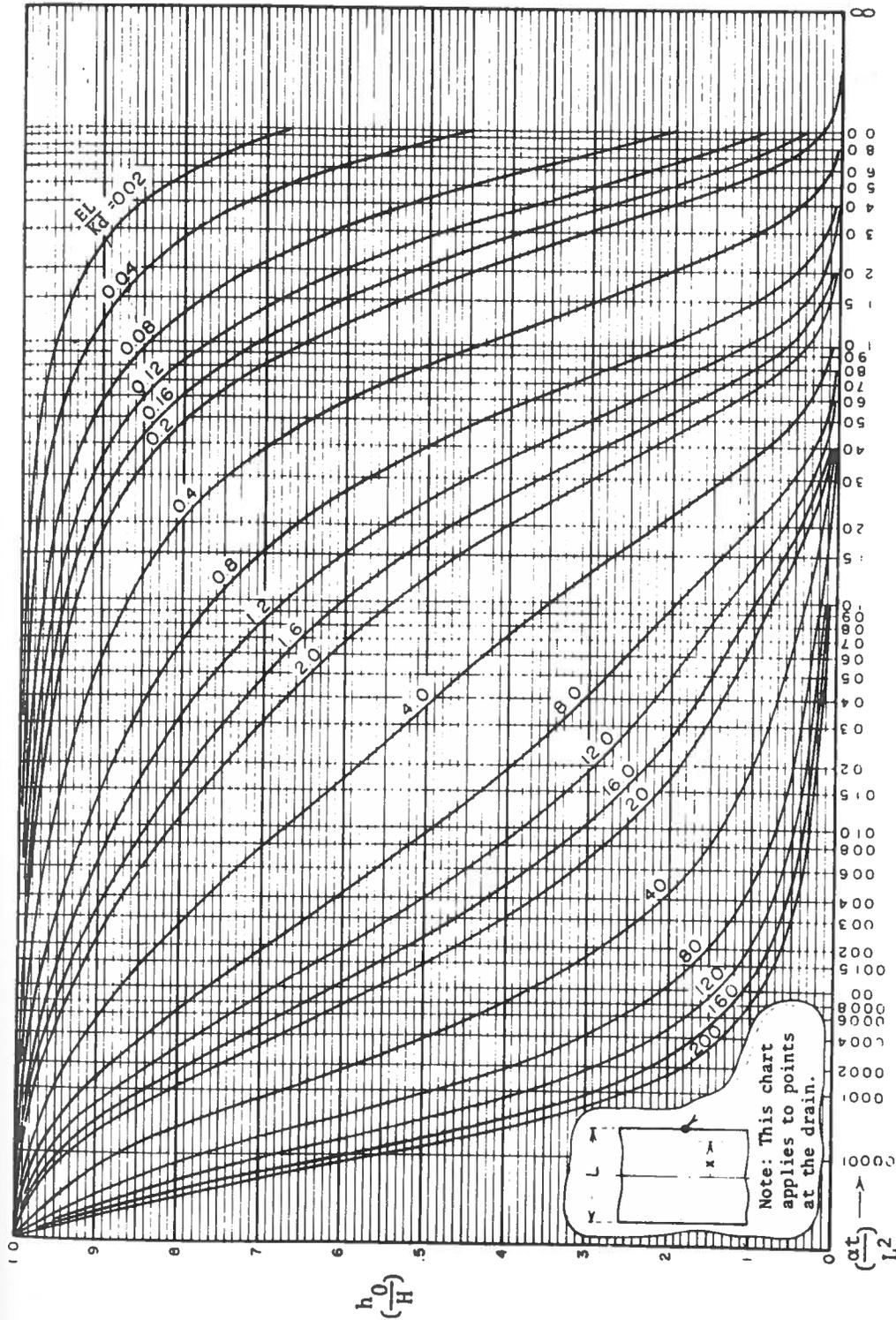


Fig. 8-7 (h_0/H) vs ($\alpha t/L^2$) taking account of local flow resistance at the drain.

or

$$[p_1 - p_2 - p_3] \cong \frac{2p_0}{\pi} \log_e \left(\frac{d}{\pi a} \right)$$

We need a ratio expressing the flow rate and the head causing the flow. Set

$$E h_0 = q \quad (8-24)$$

Where h_0 represents the head required to overcome the convergence losses and q represents the flow to unit length of drain from one side. Then:

$$E = \frac{q}{h_0} = \frac{2Kp_0}{\frac{2p_0}{\pi} \log_e \left(\frac{d}{\pi a} \right)} = \frac{\pi K}{\log_e \left(\frac{d}{\pi a} \right)} \quad (8-25)$$

As an example of the use of this result the problem whose solution is given in the paragraph on "Selection of drain spacings" will be resolved taking the convergence losses into account. In so doing, the charts prepared for an analogous problem in the flow of heat will be used. With

$$K = 10 \text{ ft/day}$$

$$\alpha = 1222.23 \text{ ft}^2/\text{day}$$

$$d = 22 \text{ ft}$$

$$Kd = 220 \text{ ft}^2/\text{day}$$

$$a = 0.5 \text{ ft}$$

$$L = 1500 \text{ ft}$$

$$\left(\frac{d}{\pi a} \right) = \frac{22}{1.5708} = 14.006$$

$$\left(\frac{d}{\pi a} \right) \gg 1$$

$$\log_e 14.006 = 2.63949$$

$$E = \frac{\pi K}{\log_e \left(\frac{d}{\pi a} \right)} = \frac{(3.1416) (10)}{2.63949} = 11.902 \text{ (ft/sec)}$$

$$\frac{EL}{KD} = \frac{(13.843) (1500)}{220} = 81.152 \text{ (Dimensionless)}$$

At the end of the irrigation season the depth of water at the drains is estimated in the following manner:

Application	Time Days	Drainable Depth (feet)	$\left(\frac{\alpha t}{L^2}\right)$	$\frac{h_0^{**}}{\left(\frac{H}{H}\right)}$	h_0 (feet)
Apr 22*	132	0.46	0.0717	0.025	0.0115
June 6	87	0.46	0.0445	0.030	0.0138
July 1	62	0.46	0.0337	0.035	0.0161
July 21	42	0.46	0.0228	0.045	0.0184
Aug 4	28	0.46	0.0152	0.055	0.0253
Aug 18	14	0.46	0.0076	0.082	0.0377
Sept 1	0	0.46	0	1.000	0.4600
	365	4.00	0.1938	0.010	<u>0.0400</u> 0.6228

*Snowmelt

**From chart of figure 8-7.

Then if water flows to the drain from both sides

$$2q = 2Eh = 2 (11.902) (0.6228) = 14.825 \text{ ft}^2/\text{day} .$$

This value is about three times as high as was obtained for this case by using the Maasland idealization. The reason for the difference is that the value computed above is a peak value whereas the Maasland value is in the nature of an average. It would be good engineering to design the drains to carry the peak flows since, otherwise, the computed drainage performance could not be obtained. It may be noted also that the additional cost of a slightly larger tile would be a small part of the cost of installing the drains.

It remains to assess the effect of the convergence losses upon the drainage performance. The following computation will provide drainable depth values at midspan which can be compared with similar values where the convergence loss was neglected.

Application	Time Days	Drainable Depth (feet)	$\left(\frac{\alpha t}{L^2}\right)$	$\frac{h}{\left(\frac{c}{H}\right)}$	$\frac{h}{\text{(feet)}}$
Apr 22*	132	0.46	0.0717	0.625	0.288
June 6	87	0.46	0.0445	0.825	0.380
July 1	62	0.46	0.0337	0.900	0.414
July 21	42	0.46	0.0228	0.960	0.442
Aug 4	28	0.46	0.0152	0.990	0.455
Aug 18	14	0.46	0.0076	1.000	0.460
Sept 1	0	0.46	0	1.000	0.460
	365	4.00	0.1938	0.210	<u>0.840</u>
					3.739

*Snowmelt

**From chart of figure 8-6 with $(EL/KD) = 81.2$.

A comparison of the results of this computation with a similar one in the paragraph on selection of drain spacings will show that the effect of local convergence on drain performance is not great. As would be expected, the computations show a slower drainage when local convergence losses are accounted for.

Return flows from irrigations

The idealizations described in this chapter can be adapted to the task of estimating the pattern of return flows supplied by deep percolations originating in irrigations. These return flows are often an important part of the water supply in irrigated areas. Figures 8-8 and 8-9 show how the conditions in a river valley can be correlated with the parallel drain idealization.

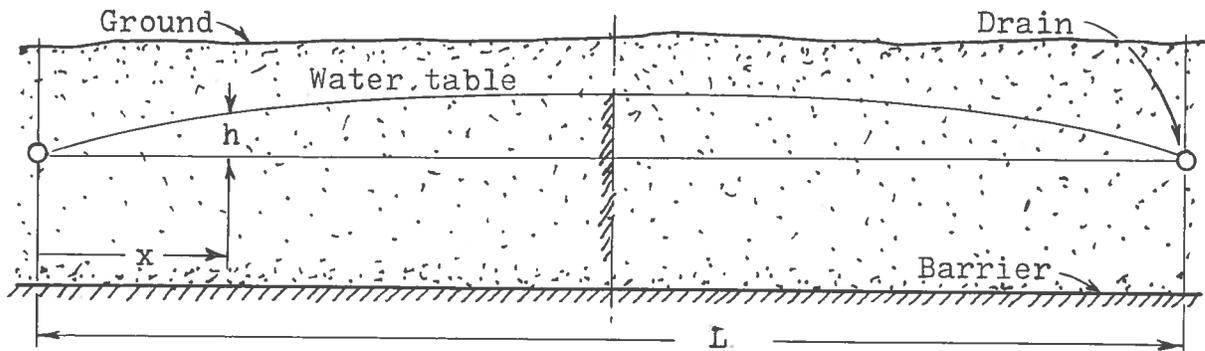


Fig. 8-8 Parallel drain idealization.

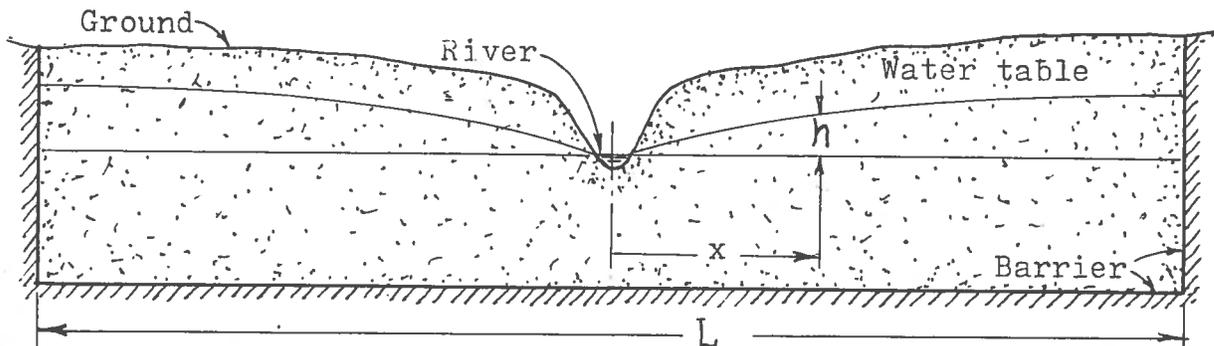


Fig. 8-9 Idealization of a river valley.

Because there is no flow across the line midway between drains, as shown in figure 8-8, the figure may be cut in two there and rearranged to bring the drains into coincidence, as shown in figure 8-9. Here the river replaces the drains. Use will be made of idealization 8-9 later. Mathematically, the idealizations of figures 8-8 and 8-9 are identical so long as L represents both the valley width and the drain spacing.

Chapter 9Stream depletion due to a well

Formula 3-1 of Chapter 3 is appropriate where a well draws water, at the constant rate Q , from an aquifer of uniform properties and of infinite extent. The presence of a flowing stream may impose a condition of no drawdown along its course. If the course of the stream can be idealized as a straight line the condition of no drawdown can be imposed by use of an image well as explained in Chapter 10. In this case the image well is a recharge well of strength $-Q$ and is located at the same distance from the stream as the pumped well and directly across the stream from it. The gradients imposed transverse to the stream by this combination can be computed from equation 3-1 and the flows produced by them can be summed along the whole stream length. In the mathematical sense this will be from $-\infty$ to $+\infty$. The depletion flow will be zero at time zero and will gradually rise toward Q as time increases.

The depletion of the stream by the well, computed in this way is given by the expression

$$\frac{q_1}{Q} = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\left(\frac{x_1}{\sqrt{4\alpha t}}\right)} e^{-u^2} du \quad (9-1)$$

For a given case this can be evaluated by use of Table 8. Details of this development are given in the paper by Glover and Balmer 1954.

Example

A well is located one mile from a stream. The aquifer properties are $KD = 0.270 \text{ ft}^2/\text{sec}$ $V = 0.17$ $\alpha = 1.59 \text{ (ft}^2/\text{sec)}$. It is desired to estimate what part of the flow of this well will be depleting the stream after the pumping has continued for three months.

With $x_1 = 5280 \text{ ft} = t \quad (3)(2628000) = 7884000 \text{ seconds}$

$$\sqrt{4\alpha t} = \sqrt{(4)(1.59)(7884000)} = 7077 \left(\frac{x_1}{\sqrt{4\alpha t}} \right) = 0.746$$

From Table 8: $\frac{2}{\sqrt{\pi}} \int_0^{0.746} e^{-u^2} du = 0.70858$

then

$$\frac{q_1}{Q} = 1 - 0.70858 = 0.29142$$

and the stream depletion at this time is about 29 percent of the well flow.

If the well had maintained a flow of $Q = 1.50 \text{ ft}^3/\text{sec}$ the stream depletion, at this time, would be

$$q_1 = (1.50)(0.29142) = 0.437 \text{ (ft}^3/\text{sec)}.$$

The pattern of stream depletion due to a well can be of interest. If equation 3-1 is written in the form:

$$s = \frac{Q}{2\pi KD} \int_{\frac{\sqrt{x^2+z^2}}{\sqrt{4\alpha t}}}^{\infty} \frac{e^{-u^2}}{u} du$$

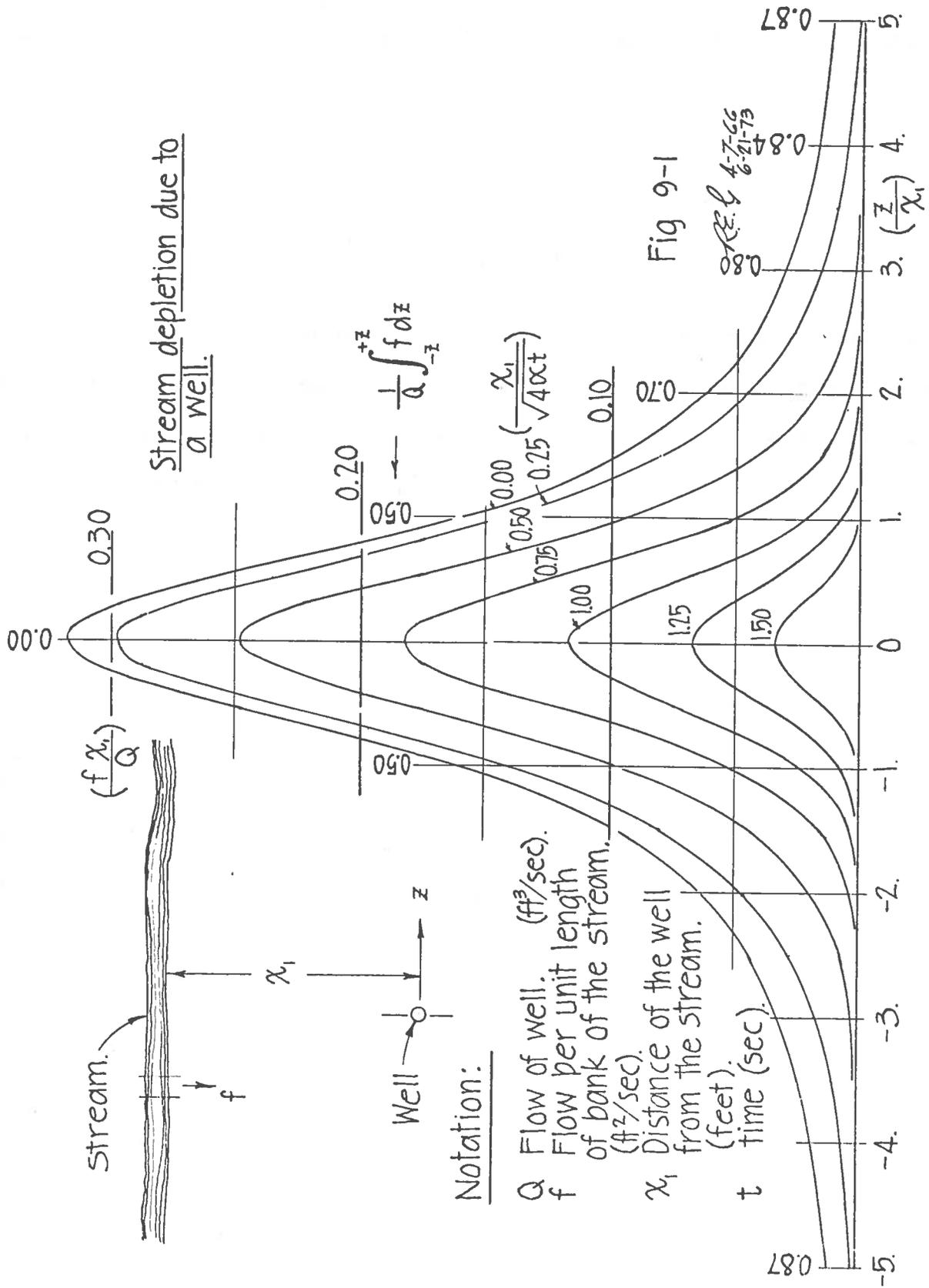
Then the cross stream gradient is

$$\frac{\partial s}{\partial x} = \frac{-Q}{2\pi KD} \frac{x e^{-\left(\frac{x^2+z^2}{4\alpha t}\right)}}{x^2+z^2}$$

The image well will produce a similar gradient. Then the flow from the stream, per unit length of the stream, will be:

$$f = 2KD \frac{\partial s}{\partial x} = \frac{Q}{\pi x_1} \frac{x_1^2 e^{-\left(\frac{x_1^2+z^2}{4\alpha t}\right)}}{(x_1^2+z^2)} \quad (9-2)$$

The chart of figure 9-1 has been prepared by use of this expression.



An ultimate steady state is reached when $(x_1/\sqrt{4\alpha t})$ becomes zero. Under these conditions the flow coming from the reach $-z$ to $+z$ is given by

$$\int_{-z}^{+z} f dz = \frac{Q}{\pi x_1} \int_{-z}^{+z} \frac{x_1^2 dz}{(x_1^2 + z^2)} = \frac{2Q}{\pi} \arctan \frac{z}{x_1} \quad (9-3)$$

The figures on the chart which read vertically show values of this integral. When the ultimate steady state is reached, for example, one-half of the flow of the well will come from a reach of the river $2x_1$ long centered on the well. A similar reach $10x_1$ long will supply 87 percent of the well flow.