Development of Crop Production Functions For Irrigation in North Central Kansas

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Summary

Crop production functions for irrigation show the relationship of crop yield and irrigation. Often, especially in economic analyses, yield needs to be calculated from the amount of applied irrigation. Crop production functions can be derived from field studies where different amounts of irrigation are applied and yields are measured. Crop production functions can also be in the form of mathematical equations that describe the diminishing return characteristic of yield resulting from more and more irrigation.

Field studies to find yields with respect to irrigation are usually conducted at research sites where a range of irrigation amounts can be applied with specialized equipment and statistical analyses can be applied to the data. Mathematical models are calibrated to field conditions and then used in other locations.

It was necessary to calculate the crop yields that would have been produced during 2005 and 2006 had additional water been available to irrigators in the Kansas Bostwick Irrigation District (KBID). Irrigation records from previous years were available by crop and the type of irrigation system, but the yields resulting from the amount of irrigation applied were not available. Yields from fully irrigated and dryland management are reported by county by the National Agricultural Statistical Service (NASS), but the yields from irrigation between the extremes are not. Likewise, Kansas State University crop performance testing in the KBID region was conducted with either fully irrigated or dryland management.

Production functions, based on a mathematical model developed by researchers at the University of Nebraska-Lincoln (UNL), were chosen to calculate crop yields over the range of irrigation that had been used by irrigators in the KBID prior to 2005. The numerical parameters needed for the UNL model were derived by the UNL researchers for counties in Nebraska, eastern Colorado and central and western Kansas. The model was used to calculate yields for north-central Kansas. The differences between yields that would have been produced in 2005 and 2006 with irrigation and yields expected when no irrigation was available were the basis for the economic analysis.

The model developed at UNL was further evaluated with field data gathered at Kansas State University's research center in Garden City. This study, conducted with six irrigation treatments, produced yield results with respect to irrigation. The production function from these data was compared with UNL's production function that used parameters derived for the Garden City location. The results from the field data and the mathematical model were very similar.

Mathematical Model

Crops generally respond positively to inputs like fertilizer and irrigation until maximum yield is obtained. However, the increment of yield gain from each increment of input declines as more inputs are added. A "diminishing return" curvilinear mathematical model often fits observed data and describes the phenomenon. In 1984, Dr. Derrel Martin, Professor, University of Nebraska (Martin et al., 1984) suggested that the Cobb-Douglas mathematical model could be used to describe grain yield response to irrigation. The equation was presented in Martin et al., 2010:

$$Y = Y_n + (Y_f - Y_n)\,[\,1 - (\,1 - D/D_f)^{1/\beta}\,] \quad \text{where } \beta = (ET_f - ET_n)/D_f \qquad (1)$$

The equation also can be written as:

$$Y = Y_n + b\,(ET_f - ET_n)\,[\,1 - (\,1 - D/D_f)^{1/\beta}\,] \text{ where } b\,(ET_f - Et_n) = (Y_f - Y_n) \qquad (2)$$

The second form of the equation was used in developing the crop production function for north central Kansas

"Y" is the unknown grain yield (dependent variable) that is derived with equation 2.

"D" is the amount of irrigation (independent variable) that is delivered to the field.

"D_f" is the amount of irrigation required to produce maximum yield. **Net irrigation requirement** (NIR) is the infiltrated irrigation water that is necessary to produce maximum yield. It depends on geographic location (particularly precipitation) and crop. NIR requirement varies with rainfall probabilities; hence, location is important. D_f can be derived from NIR by dividing NIR by application efficiency (AE).

"Y_n" is the non-irrigated yield that is produced from precipitation only. Values for Y_n are as a result of growing a summer row crop that was not irrigated the year before. County yield averages for dryland crops, reported by NASS, include crops that may have followed the same or another row crop or the crop may have followed winter wheat. The typical 3-year dryland crop rotation across the Republican River Basin is winter wheat followed by sorghum or corn followed by fallow from harvest of sorghum or corn until wheat planting. Dr. Martin derived values for Y_n from a crop simulation model explained later in this report.

"Y_f" is the maximum yield that a crop can produce if unrestricted by inputs such as fertilizer and chemicals for weed control and insect control.

"b" is the slope of the yield-evapotranspiration (ET) function that has been proven to follow a linear model by many field studies. ET is the combination of the water consumed by the crop, transpiration (T) and water evaporated directly from the soil surface (E). The form of the yield-ET function is (Martin et al., 2010):

$$Y = Y_n + b\,(ET - ET_n) \qquad (3)$$

"ET_f- ET_n" or **"ET-increase"** (ET_{inc}). ET_f is the amount of water used by a fully irrigated crop for maximum yield. ET_n is the amount of water used by a non-irrigated crop. ET_{inc} is the difference between ET_f and ET_n, which is the amount of water used by the crop to produce yield. Yield is grain in the case of grain crop and forage in the case of forage crops such as alfalfa.

"β" (beta) is the value for the exponent in equations 1 and 2. It influences the curvilinear shape or the yield response to irrigation and is related to application efficiency (AE), the ability of the irrigation system to deliver water to the soil surface. Irrigation systems cannot deliver all of the pumped or diverted water to the soil surface with complete spatial uniformity so different irrigation systems, particularly sprinkler versus furrow irrigation, deliver water more or less efficiently.

$$\beta = AE\,(ET_{inc}/NIR) \qquad (4)$$

Parameters for Cobb-Douglas Model

Dr. Martin has determined the numerical values of the parameters for the Cobb-Douglas equation for corn and soybean from executions of CROPSIM, a simulation model, based on results from field research and mathematical descriptions of irrigation delivery and crop development (Martin et al,. 2010). Dr. Martin has described CROPSIM as a soil water balance of inputs to a soil volume and outputs from that same soil volume. He derived parameters by county for most of Nebraska, western and central Kansas, and eastern Colorado where irrigated crops are grown (Supalla, 2011). The parameters for Republic CO, Kansas were used across the KBID region because the preponderance of the acreage in KBID is in Republic, CO.

Table 1. Parameters for Cobb-Douglas equation for Republic CO, Kansas as applied to north-central Kansas (parameters in bold were derived by Martin with CROPSIM).

Crop	System	NIR	ET_{inc}	Y_n/Y_f	Y_f	b	Y_n	AE	β
		in	in		bu/ac	bu/ac-in	bu/ac	%	
Corn	Center Pivot	**10.1**	**7.5**	**0.54**	**182**	**11.2**	**98**	**85**	**0.63**
Corn	Furrow	**10.1**	**7.5**	**0.54**	**182**	**11.2**	**98**	**60**	**0.45**
Soybean	Center Pivot	**8.6**	**5.7**	**0.68**	**63**	**3.5**	**43**	**85**	**0.56**
Soybean	Furrow	**8.6**	**5.7**	**0.68**	**63**	**3.5**	**43**	**60**	**0.40**
Sorghum	Center Pivot	7.4[1]	5	0.76	134[2]	6.4	102[2]	**85**	0.57
Sorghum	Furrow	7.4[1]	5	0.76	134[2]	6.4	102[2]	**60**	0.41
					ton/ac	ton/ac-in	Ton/ac		
Alfalfa	Center Pivot	16[1]	12	0.60	6.5[3]	0.2	3.9[4]	**85**	0.64

[1]From USDA Natural Resources Conservation Service Kansas Irrigation Guide.

[2]From Kansas State University Performance Test Data & National Agricultural Statistical Service (NASS).

[3]From consultation with Scott Staggenborg, Kansas State University Agronomist

[4]From NASS

The parameters for sorghum and alfalfa (ET_{inc}, and b) were derived by Dr. Klocke based on data he collected in field studies at Kansas State University's Research Center at Garden City, Kansas which were adapted to north central Kansas. Yield and ET data for sorghum and alfalfa were collected in the field study. From these data, b and ET_{inc} could be calculated. AE values were the same as those used by Martin. The calculation of β used NIR data from the USDA Soil Conservation Service, AE values from Martin, and ET_{inc} from Klocke's data.

Calculated Yields

Parameters from table 1 were applied to equation 2 to calculate yields for each amount of applied irrigation in table 2 and figures 1-3.

Table 2. Yield response to irrigation as calculated with equation 2.

Applied Irrigation	Corn CP[1]	Corn Furrow	Soybean CP	Soybean Furrow	Sorghum CP	Sorghum Furrow	Alfalfa CP
inches	bu/ac	bu/ac	bu/ac	bu/ac	bu/ac	bu/ac	tons/ac
0	98	98	43	43	102	102	3.9
1	109	109	46	46	108	108	4.1
2	120	119	49	49	114	113	4.3
3	129	128	52	52	119	118	4.5
4	139	136	54	54	123	122	4.7
5	147	144	57	56	127	125	4.9
6	155	151	59	57	130	128	5.1
7	162	157	60	59	132	130	5.2
8	168	162	61	60	134	132	5.4
9	173	167	62	61	134	133	5.6
10	178	171	63	62	134	133	5.7
11	181	174	63	62	134	134	5.8
12	182	177	63	62	134	134	6
13	182	179	63	63	134	134	6.1
14	182	180	63	63	134	134	6.2
15	182	181	63	63	134	134	6.3
16	182	182	63	63	134	134	6.4
17	182	182	63	63	134	134	6.4
18	182	182	63	63	134	134	6.5

[1]CP=Center Pivot

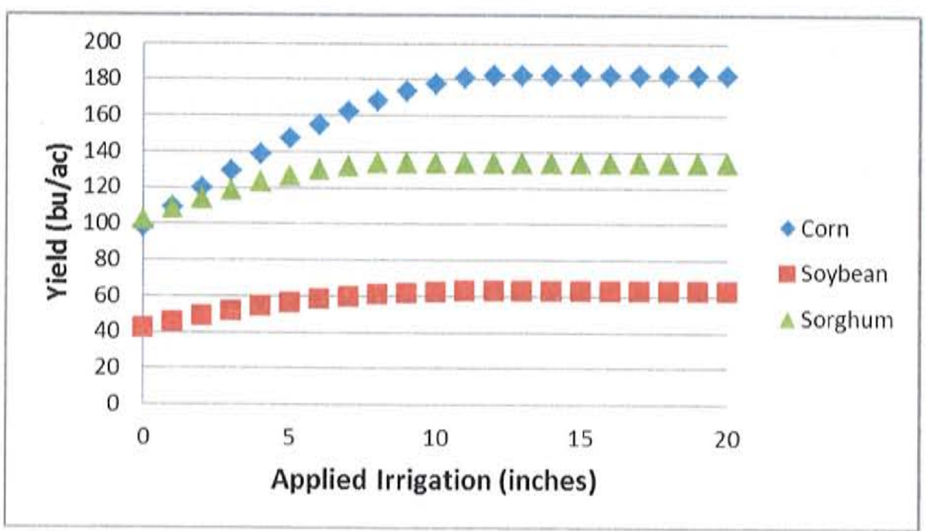

Figure 1. Yield response to irrigation for corn, soybean, and sorghum using center pivots.

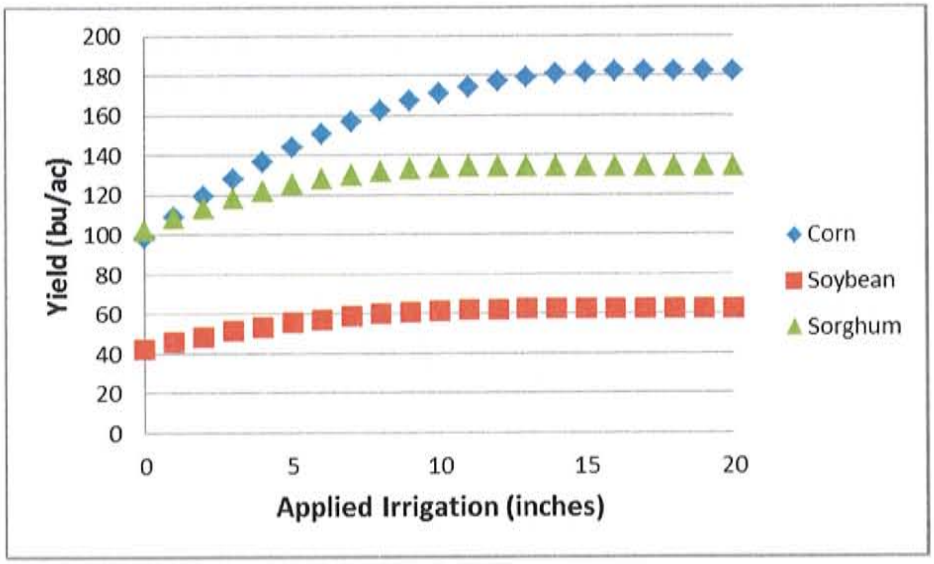

Figure 2. Yield response to irrigation for corn, soybean, and sorghum using furrow irrigation.

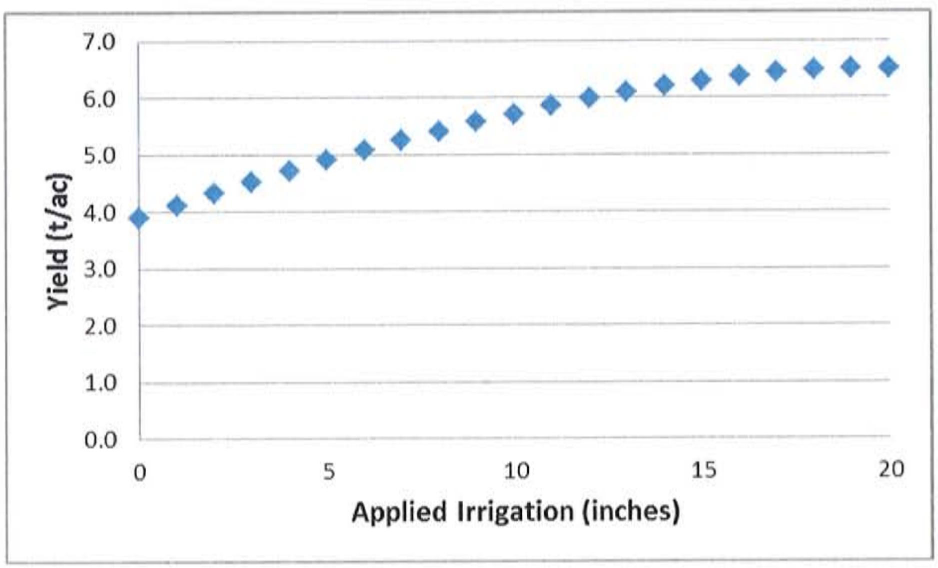

Figure 3. Yield response to irrigation for alfalfa using center pivot irrigation.

Field Results Compared with Cobb-Douglas Equation

To measure yield responses to irrigation in the field, irrigation needs to vary across plots from no irrigation or very little irrigation to irrigation for maximum yields. Because the yield response to irrigation is curvilinear, several levels of irrigation need to be applied to describe the diminishing yields as irrigation increases. Dr. Klocke has conducted field experiments to gather corn yield data for six levels of irrigation during 2005-2009 in Garden City, Kansas (Klocke et al., 2011). The measured yields for each irrigation amount for each year were divided by the maximum yield for that year to produce "relative yields". The relative yield for the maximum irrigation for every year was 1 and the remaining 5 irrigation treatment yields were fractions of 1. Relative yields can be compared across years better than the measured yields because the year-to-year differences in factors that affect yield except irrigation can be minimized.

A statistical regression of the field data for corn (relative yield versus irrigation) was derived in the form of a quadratic equation (Klocke et al., 2011):

$$\text{Relative Yield} = -0.0026\,(NI)^2 + 0.084\,(NI) + 0.33 \qquad (5)$$

where NI = net irrigation in inches.

The net irrigation results were converted to applied irrigation and the results from equation 5 are in figure 4 as the "KSU" data points. The parameters for the Cobb-Douglas equation for Garden City, as determined by Dr. Martin, produced the relative yield results in figure 4 denoted by the C-D data points.

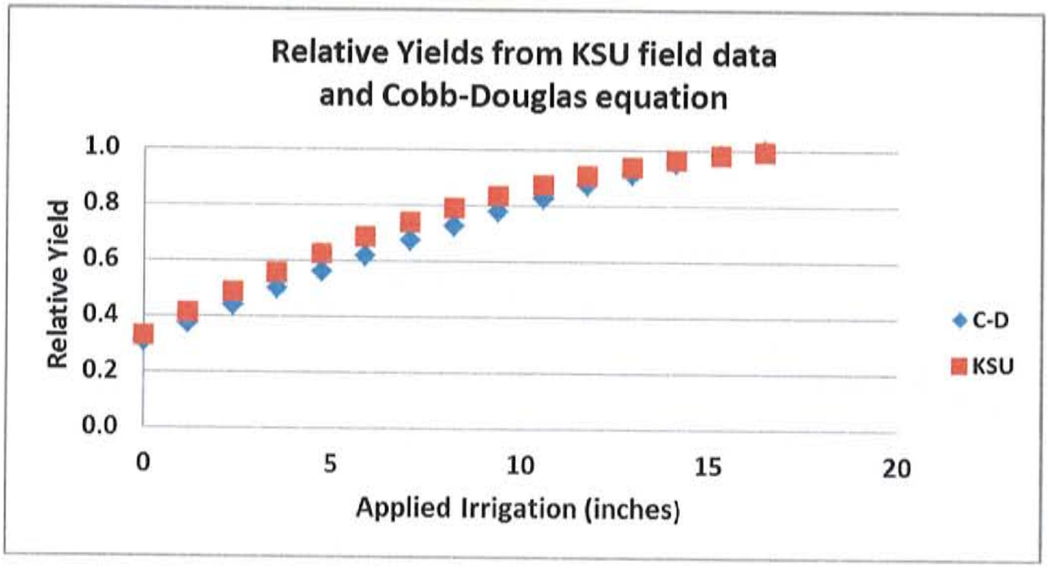

Figure 4. Relative yield for corn versus applied irrigation from Cobb-Douglas equation (C-D) and field data (2005-2009) from Kansas State University at Garden City (KSU).

The results in figure 4 show that the Cobb-Douglas equation matched the field data very well. When the value for NI in equation 5 was zero, Y_f/Y_n was calculated to be 0.33. Martin determined that Y_f/Y_n was 0.31 using CROPSIM for Garden City. Also, when Relative Yield was in equation 5 was 1, NI was calculated to be 14 inches. This corresponds to the NIR calculated by Martin which was 13.8 inches for Garden City.

References

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Martin, D.L., D.G. Watts, and J.R. Gilley. 1984. Model and production function for irrigation management. Journal of the Irrigation Drainage Division of the American Society of Civil Engineers. 110 (4):149-164.

Martin, D.L., R.J. Supalla, C.L. Thompson, B.P. McMullen, G.W. Hergert, and P.A. Burgener. 2010. Advances in deficit irrigation management. 5th National Decennial Irrigation Conference. American Society of Agricultural and Biological Engineers. Publication No. 711P0810cd. Phoenix, AZ. Dec. 5-8, 2010.

Supalla, R.J. 2011. Procedures for adjusting APH when implementing a deficit irrigation insurance practice. April 15, 2011.

Appendix

Testimony by Dr. Klocke in the last 4 years: One deposition, in *Spear T Ranch, Inc. v. Knaub, et al.*, Morrill Co., Neb., Case No. Ci03-16.

Compensation to be paid for Dr. Klocke's study and testimony in this case: $125/hour.