Exhibit 9

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Selecting a sequence of historical years to represent future scenarios

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Precipitation statistics over the active RRCA groundwater model domain and for subareas corresponding to the three states of Colorado, Kansas and Nebraska are described in the form of cumulative probability distributions for the simulation period beginning in 1918, which has been updated to include 2006. These distributions are used as a basis for selecting random sequences of historical years from the range 1990 to 2005 to represent future scenarios. Random sequences are drawn directly from a cumulative probability distribution for years 1918-2006 using a procedure known as the transformation method. The procedure is illustrated with precipitation distributions for the entire active model area and for the Kansas component of the model area.

The RRCA groundwater model simulates historical years 1918 through 2005, and is currently being updated to include 2006. Future scenarios of interest to Kansas will be based on historical years beginning with 1990, the first year for which water use reports are considered to be highly reliable. Adjustments to reported water use for historical years 1990 through 2006 have been calculated to represent current conditions of irrigated return flow based on system type, water right development and metered water use reporting.

Annual precipitation data for the RRCA model stations has been updated for 2006, and averages over the model domain and for each state within the model domain have been computed. Average precipitation and corresponding probability of occurrence for recent years and for 2006 are listed in Table 1. Annual values for years 1990-2006 are listed in Table 2 (below). Fig. 1 shows cumulative probability distributions for annual precipitation averaged over model cells within Colorado, Kansas and Nebraska, and for the entire model domain for years 1918-2006. Symbols identify recent years 1990-2006, and highlighted symbols identify 2006 values.

Table 1. Summary of precipitation and probability of occurrence for recent periods and 2006.

	Annual precipitation (inches)				Probability of occurrence of lower annual rainfall P(precip <precip*), 1918-2006			
	CO	KS	NE	model	CO	KS	NE	model
avg(1980-1989)	16.78	20.02	22.33	20.01	0.5889	0.5222	0.6667	0.5778
avg(1990-1999)	17.69	21.48	22.89	20.94	0.6667	0.7111	0.6778	0.7111
avg(2000-2006)	14.66	18.41	19.94	17.93	0.4111	0.3778	0.3778	0.3667
avg(1990-2006)	16.44	20.21	21.67	19.70	0.5667	0.5444	0.5778	0.5556
2006	13.72	21.34	22.07	19.42	0.2889	0.7000	0.6333	0.5111

A preliminary step in building future scenarios is to select a sequence of historical years from the available set, with adjustments to present conditions for return flow, water right development and metered reporting of water use. For example, a 48-year future scenario for years 2006 through 2053 can be constructed by repeating the chronological sequence of years 1990-2005 three times. Alternatively, adjusted years from the period 1990-2005 can be randomly drawn to represent future years. However, the sequence should be hydrologically representative of historical

conditions over the simulated period beginning in 1918, allowing for possible changes to these conditions in the future such as long-term climatological projections.

Table 2. Precipitation and probability of occurrence[1] for years 1990-2006.

	Annual precipitation averaged over state subareas and active model area				Fraction of years (1918-2006) with less than or equal precipitation			
Year	CO	KS	NE	prcp	CO	KS	NE	prcp
1990	16.867	20.289	20.070	19.210	0.60000	0.57778	0.40000	0.50000
1991	17.774	21.074	21.273	20.197	0.67778	0.66667	0.53333	0.63333
1992	17.806	23.278	22.877	21.533	0.68889	0.81111	0.67778	0.73333
1993	15.139	27.628	31.589	25.592	0.46667	0.95556	0.98889	0.95556
1994	15.219	18.966	20.649	18.552	0.47778	0.43333	0.43333	0.44444
1995	21.721	21.659	22.084	21.847	0.96667	0.72222	0.64444	0.76667
1996	19.670	22.915	27.089	23.641	0.86667	0.77778	0.88889	0.86667
1997	14.280	18.972	20.742	18.320	0.36667	0.44444	0.45556	0.42222
1998	16.928	20.235	19.379	18.936	0.61111	0.56667	0.35556	0.48889
1999	21.459	19.751	23.128	21.594	0.95556	0.52222	0.71111	0.74444
2000	13.340	16.498	19.488	16.769	0.26667	0.21111	0.37778	0.26667
2001	15.538	20.485	21.797	19.576	0.50000	0.60000	0.62222	0.54444
2002	10.307	12.077	12.952	11.914	0.03333	0.03333	0.01111	0.01111
2003	14.436	14.792	18.672	16.238	0.40000	0.13333	0.26667	0.22222
2004	17.634	20.606	23.293	20.818	0.65556	0.62222	0.72222	0.71111
2005	17.648	23.059	21.295	20.787	0.66667	0.80000	0.54444	0.70000
2006	13.722	21.340	22.069	19.424	0.28889	0.70000	0.63333	0.51111

[1]Fraction of historical record (1918-2006) with less than or equal precipitation than current year.

One approach to doing this is a two-step procedure devised and demonstrated by Steve Larson, SSPA, on the basis of calculated recharge. The approach is to first draw a large number of random sequences of years between 1990 and 2005 on the basis of a uniform probability distribution, and then to select those sequences whose recharge distributions are similar to the one for historical years 1918-2005. These sequences can then be used directly, repeated or combined to represent future scenarios. A more detailed description of this approach is provided in an Appendix.

Monte Carlo selection using the transformation method

A representative sequence of years from the period 1990-2005 (or through 2006) can also be selected that is drawn directly from a distribution for historical years 1918 to the present using the transformation method (Press et al., 1986). The technique is described on the basis of annual precipitation, although it could alternatively be based on recharge.

Fig. 2 shows the cumulative probability distribution for annual precipitation 1918-2006 averaged over the active RRCA model domain. The cumulative distribution is denoted by $F(p) \equiv P(p \leq p^*)$, the probability that annual precipitation, p is less than or equal a specified value, p^*. (Equivalently, exceedance $E = 1 - P$.) The procedure, illustrated in Fig. 2, is as follows.

First, a a random number generator is used to draw a number, x, between 0 and 1 from a uniform distribution. The uniform deviate, x, is then transformed by evaluating the inverse of the cumulative probability distribution, $F^{-1}(x)$, as illustrated by Fig. 2. The resulting value of precipitation, p, has been randomly drawn from the cumulative probability distribution, $F(p)$. This approach is known as the transformation method (Press et al., 1986).

To apply the above procedure, the inverse function, $F^{-1}(x)$, is evaluated numerically by interpolating along the cumulative distribution for Kansas precipitation, and selecting the year corresponding to the probability closest to the uniform deviate, x. The spreadsheet calculation uses Excel's Match and Index functions.

The transformation method gives randomly drawn values of precipitation with the correct distribution for years 1918-2006. The procedure is adapted to select years between 1990 and 2005 by approximating the cumulative distribution using only values of precipitation and corresponding percentiles for years 1990-2005 (Fig. 3). After applying the above procedure to draw a random value of precipitation from the distribution based on years 1918-2006, the approximating year from the subset 1990-2005 is referenced instead of the actual historical year. This is demonstrated by drawing a sequence of 1000 years from the distribution for average model precipitation. restricted to years 1990-2005. Fig. 4 shows two sets of values of randomly drawn precipitation: first, the values drawn from the original distribution for years 1918-2006; and second, the corresponding values approximated by years 1990-2005.

The transformation method for selecting a random sequence of years from a distribution has been described in terms of annual precipitation averaged over the active RRCA model domain. For the purpose of developing Kansas scenarios, the average model distribution may be preferable to the Kansas distribution because the Colorado and Nebraska components of precipitation are represented more accurately than they would if selections were based on the Kansas component of precipitation. Fig. 1 shows, for percentiles less that 80 percent, that this distribution is generally within one-half inch of the distribution of the Kansas component.

Fig. 5 again shows the precipitation corresponding to the sequence of years selected from the range 1990-2005 based on the cumulative distribution for the active model, as shown in Fig. 4. Fig. 5 also shows the resulting precipitation for the Colorado, Kansas and Nebraska, and the historical distribution for each. The comparison shows that the precipitation corresponding to the generated sequence of years appears to approximate the historical distribution satisfactorily for the four cases of precipitation averaged over the active model area and the three state subareas. This suggests that future scenarios for Kansas based on these sequences of years would be representative of historical precipitation.

KSARB 0000918

References

Press, W.H., B.P. Flannery, S.A. Teukolsky and W.T. Vetterling, 1986. Numerical Recipes—The Art of Scientific Computing. Cambridge University Press, New York.

Appendix.

The technique devised by Steve Larson, SSPA, is illustrated here on the basis of precipitation instead of recharge. Each randomly chosen year is based on a number, x, between 0 and 1 that is drawn from a uniform distribution. A linear transformation is then applied to assign a corresponding year between 1990 and 2005, $y(x) = \mathrm{int}(y_0 + nx)$, where $y_0 = 1990$ and $n = 16$. For 1000 random numbers, Fig. A1 shows that this produces a uniform distribution of years. Fig. A2 compares the distribution of precipitation corresponding to the years selected by this procedure with the historical precipitation distribution for years 1918. The next step in Steve's procedure is to find 16-year sequences of randomly selected years whose distributions of precipitation are sufficiently similar to the historical distribution shown in Fig. A2, and use those sequences to represent future scenarios.

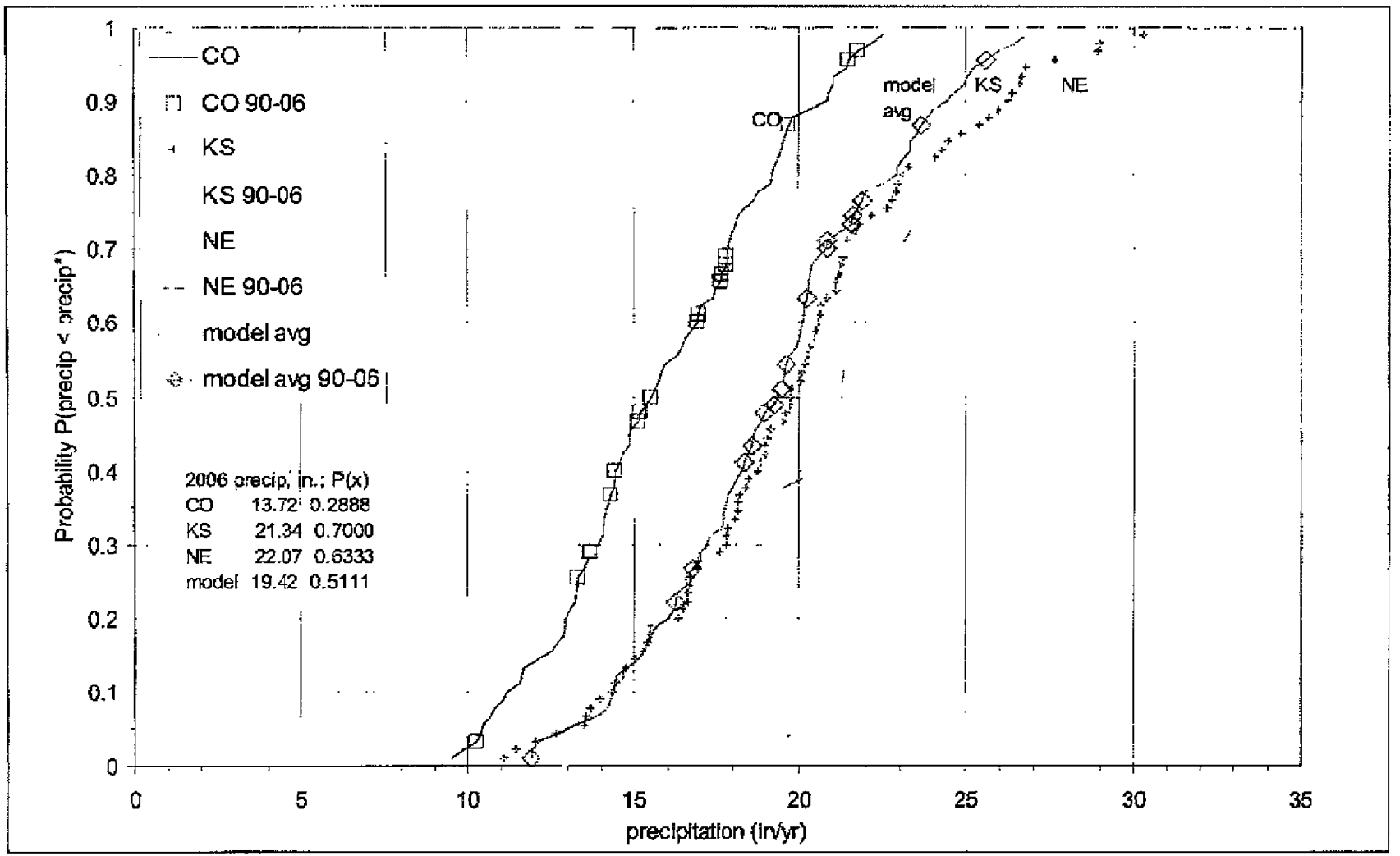

Fig. 1. Cumulative probability distributions for annual precipitation averaged over model domain within CO, KS, NE and entire over model domain for years 1918-2006 (in/yr); symbols are shown for years 1990-2006, with symbols highlighted for 2006. Chart ChProb_historical_prcp_Fig.1 in YearSelection.xls]

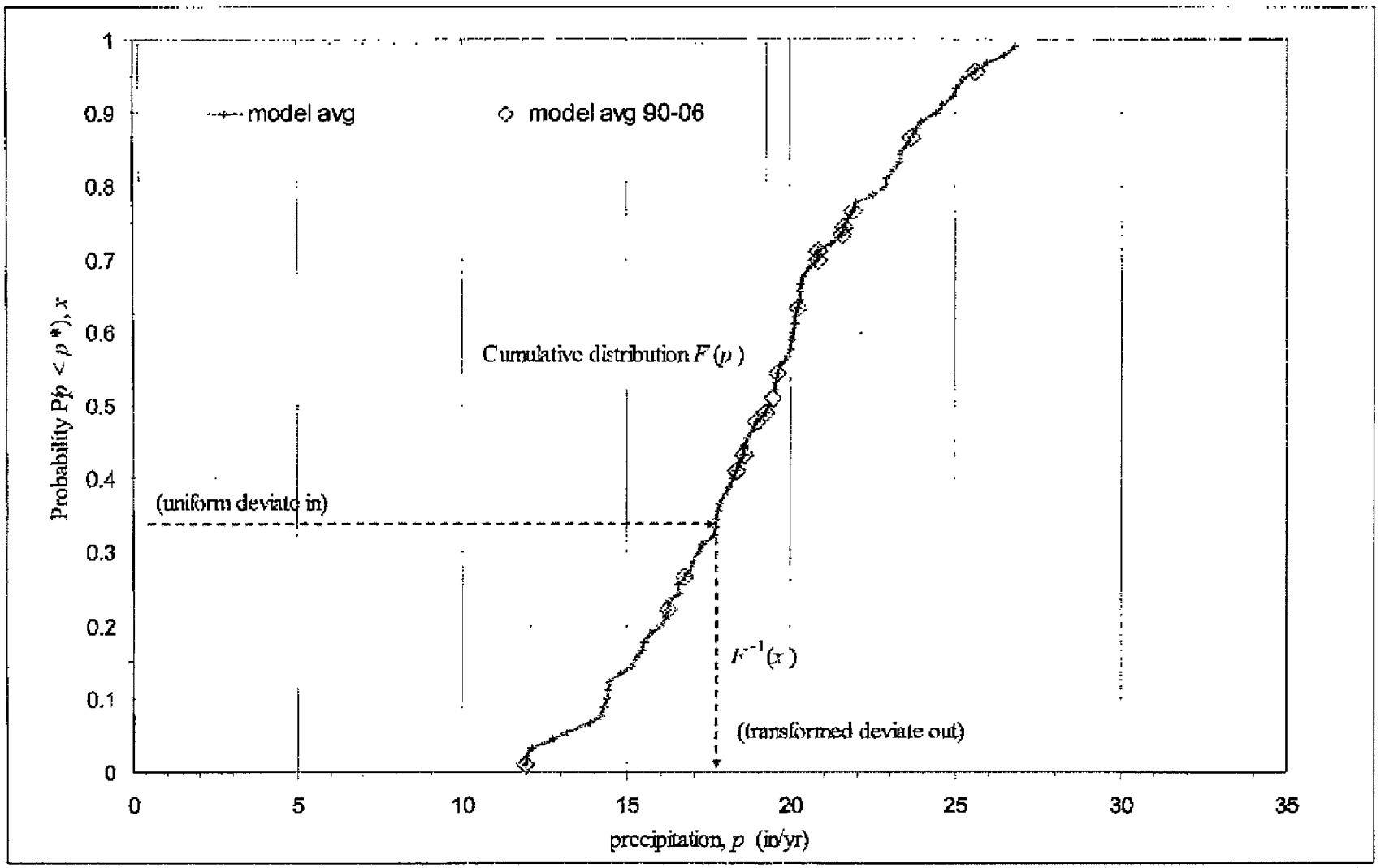

Fig. 2. The transformation method for drawing randomly from the cumulative probability distribution $F(p)$ for precipitation averaged over the entire active model area: given a random number, x, drawn from a uniform distribution between 0 and 1 (using a random number generator) as shown along the vertical axis, evaluate the cumulative distribution's inverse, $F^{-1}(x)$, to give the corresponding annual precipitation. [Chart ChModel_historical_prcp_Fig.2]

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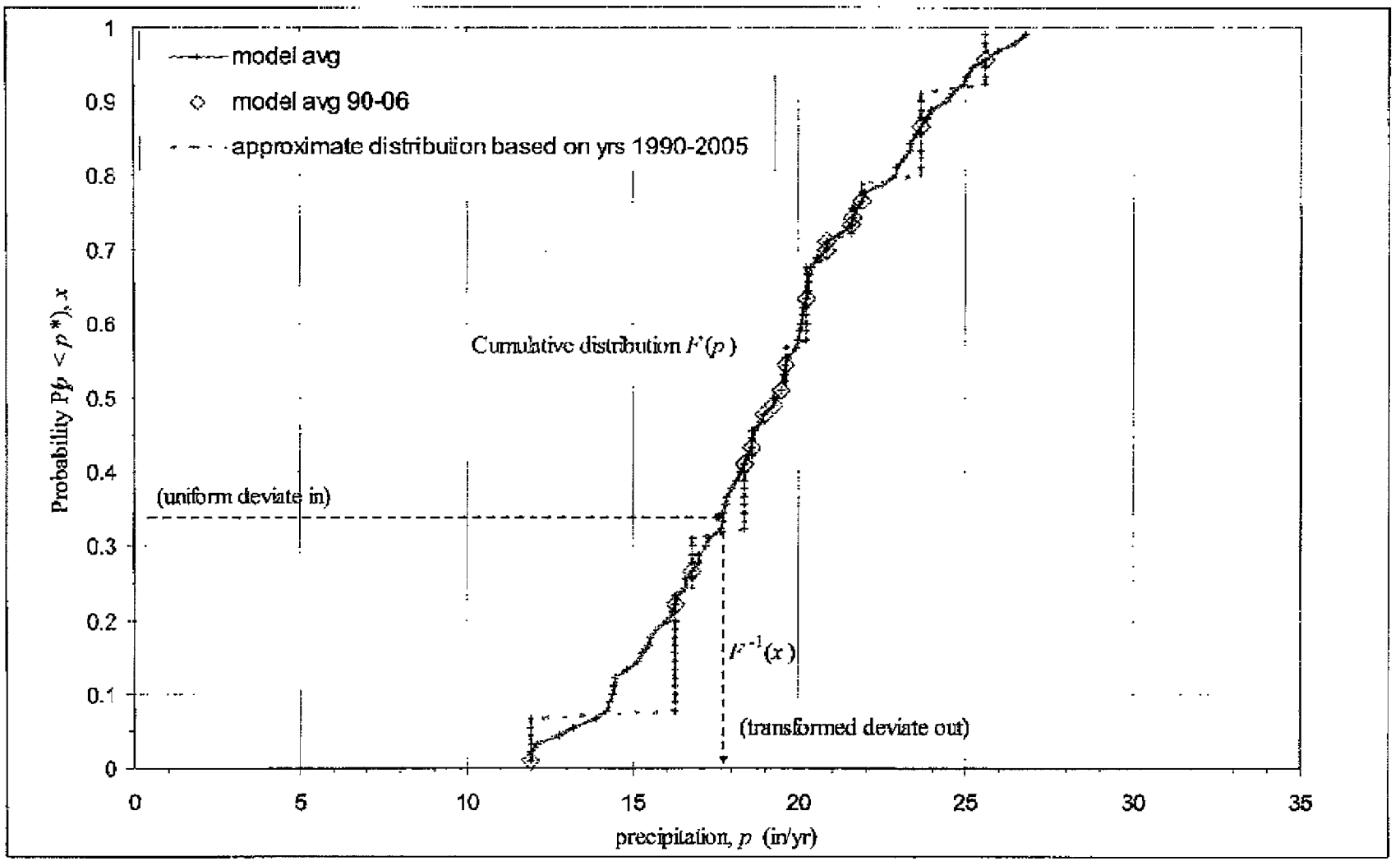

Fig. 3. After evaluating the inverse cumulative distribution for a random value of precipitation, the year with the closest historical value of precipitation and the approximating year from the period 1990-2005 are determined. [Chart ChModel_historical_prcp_Fig.3]

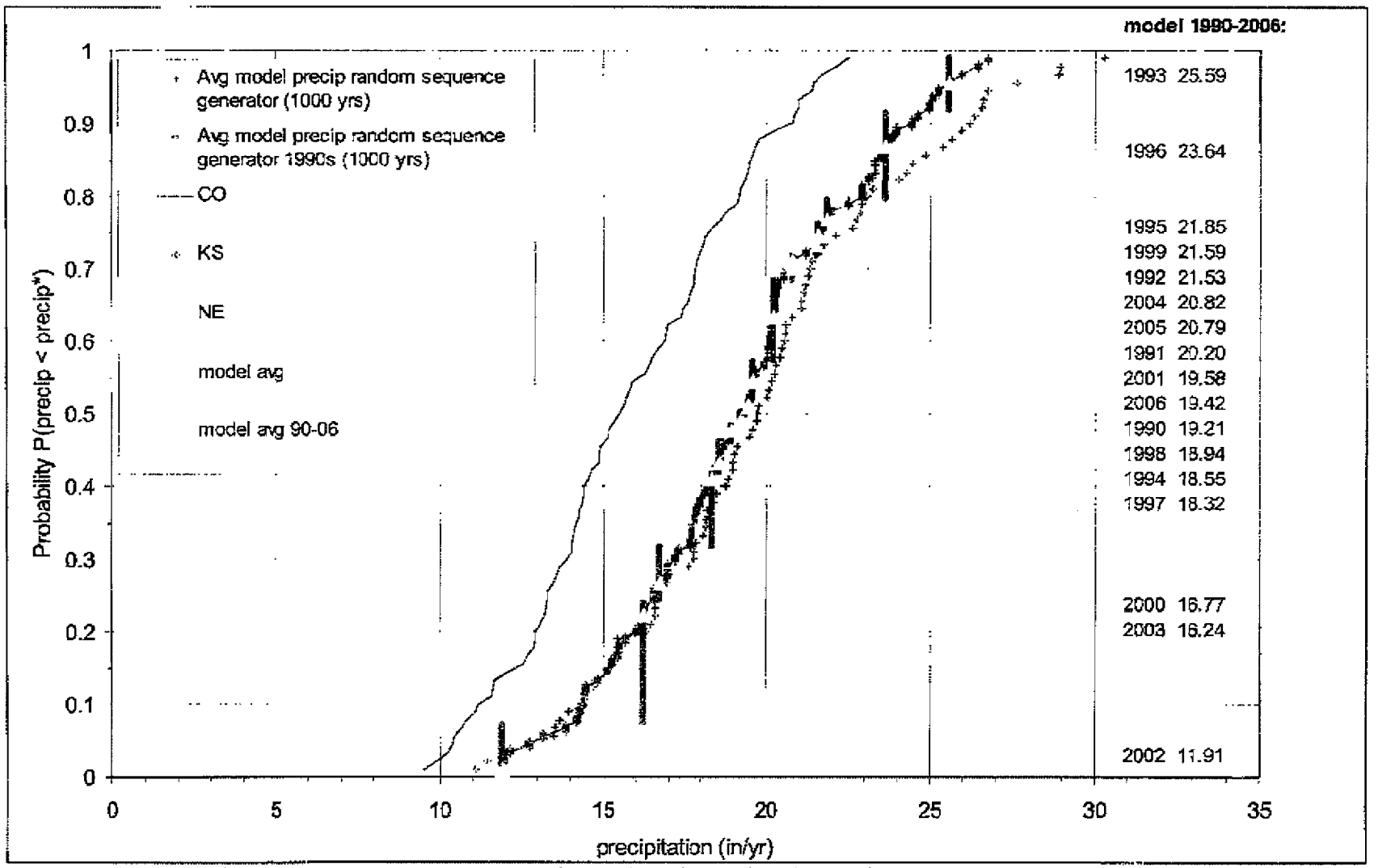

Fig. 4. Precipitation for randomly selected sequence of years based on a distribution for precipitation averaged over the entire active model area. "+": precipitation for selected years 1918-2006; "o": precipitation for selected approximating years 1990-2005. Precipitation for years 1990-2006 averaged over model area is listed at right at approximate location along vertical axis. [Chart ChModel_random_sequence_Fig.4; calculated in sheet model_sequence_generator in YearSelection.xls.]

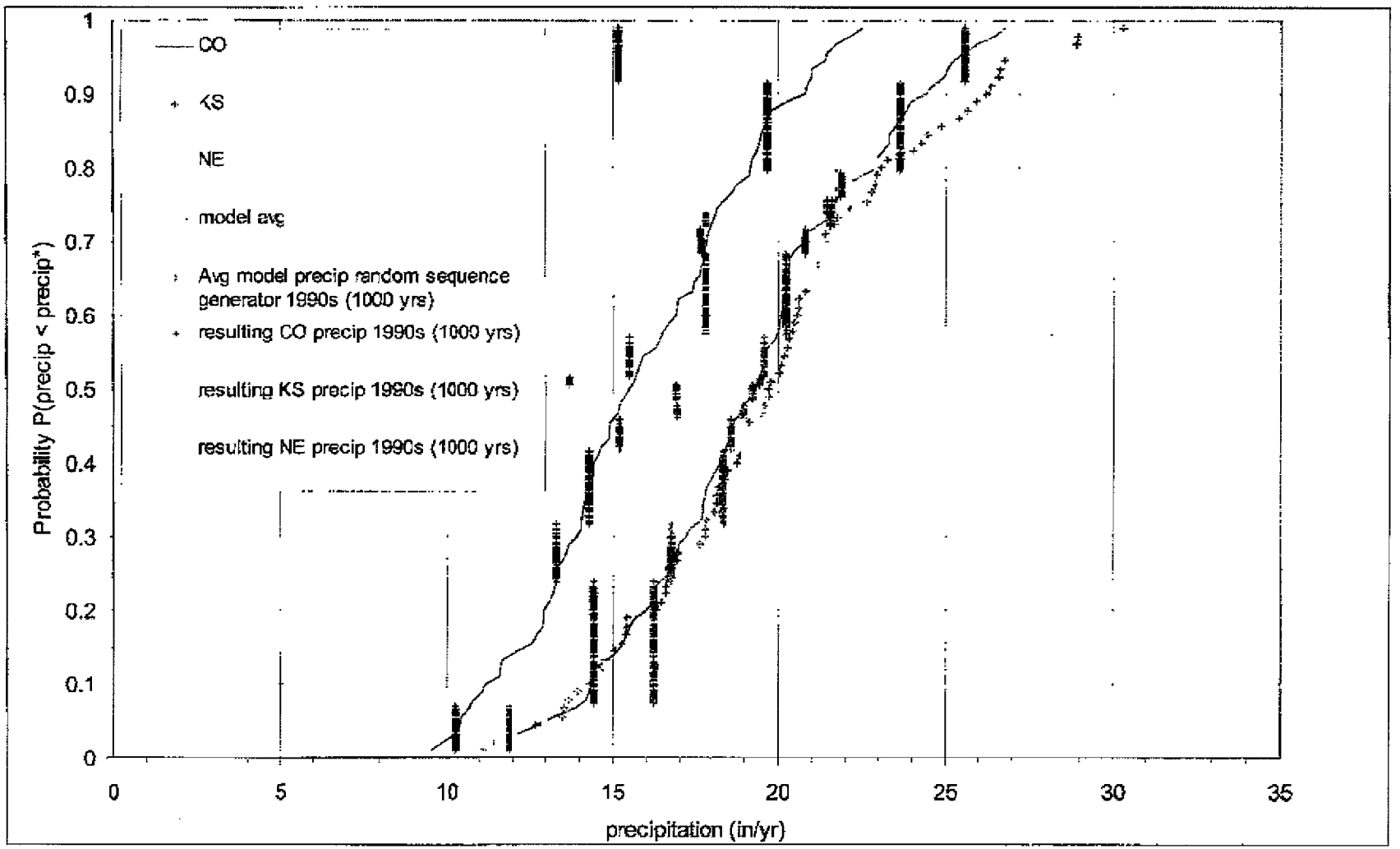

Fig. 5. Precipitation for randomly selected sequence of 1000 years from 1990-2005 based on precipitation distribution for years 1918-2006 (in/yr) spatially averaged over entire active model area, and the resulting distributions for Colorado, Kansas and Nebraska precipitation. [Chart Chmodel_random_sequence_Fig.5; calculated in sheet model_sequence_generator in YearSelection.xls, I:\RRCA_GM\KSData\ForScenarios\MonteCarlo\]

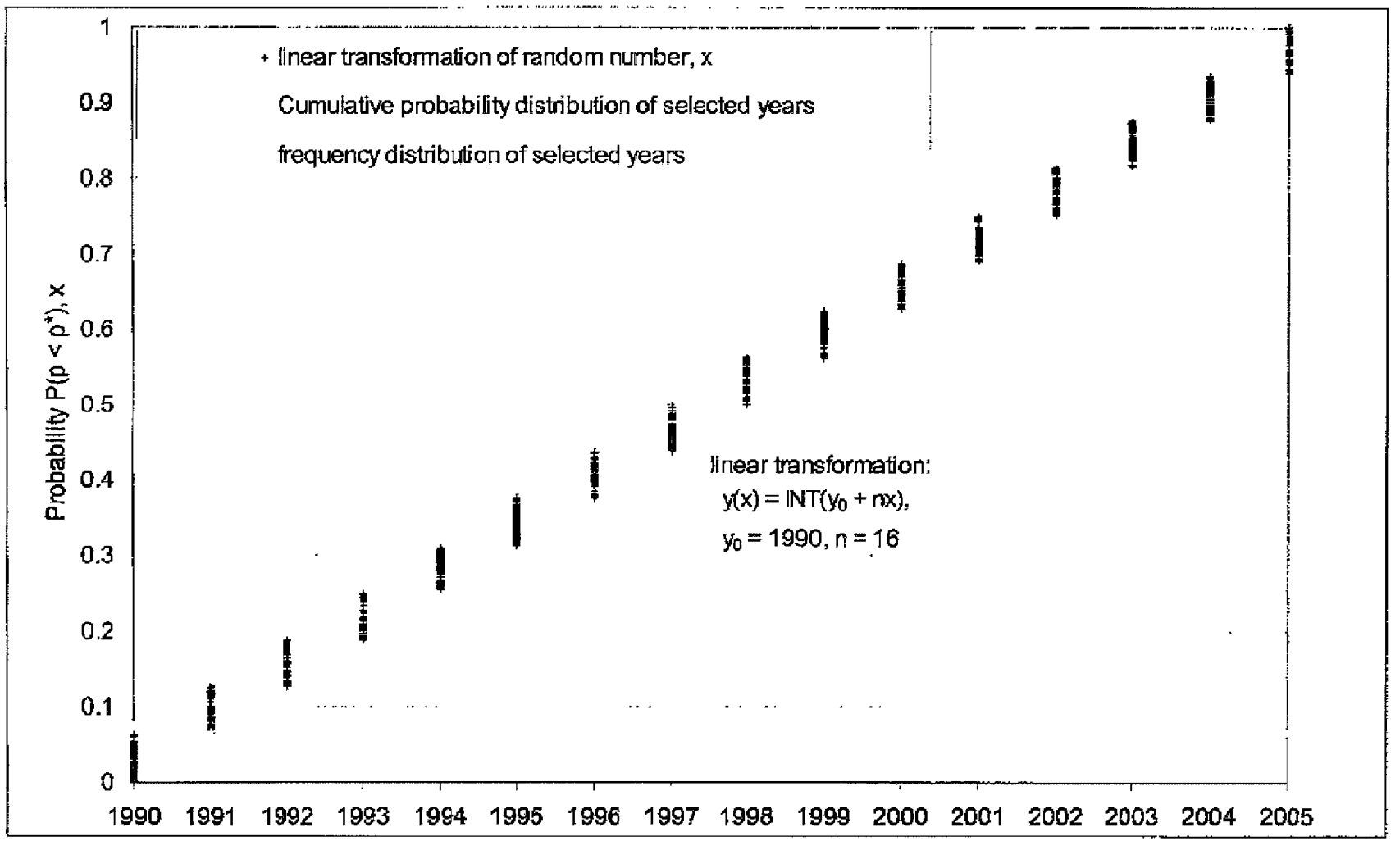

Fig. A1. Linear transformation of uniform distribution from the range (0,1) to years 1990-2005. [from sheet linear_transformation in YearSelection.xls, I:\RRCA_GM\KSData\ForScenarios\MonteCarlo\]

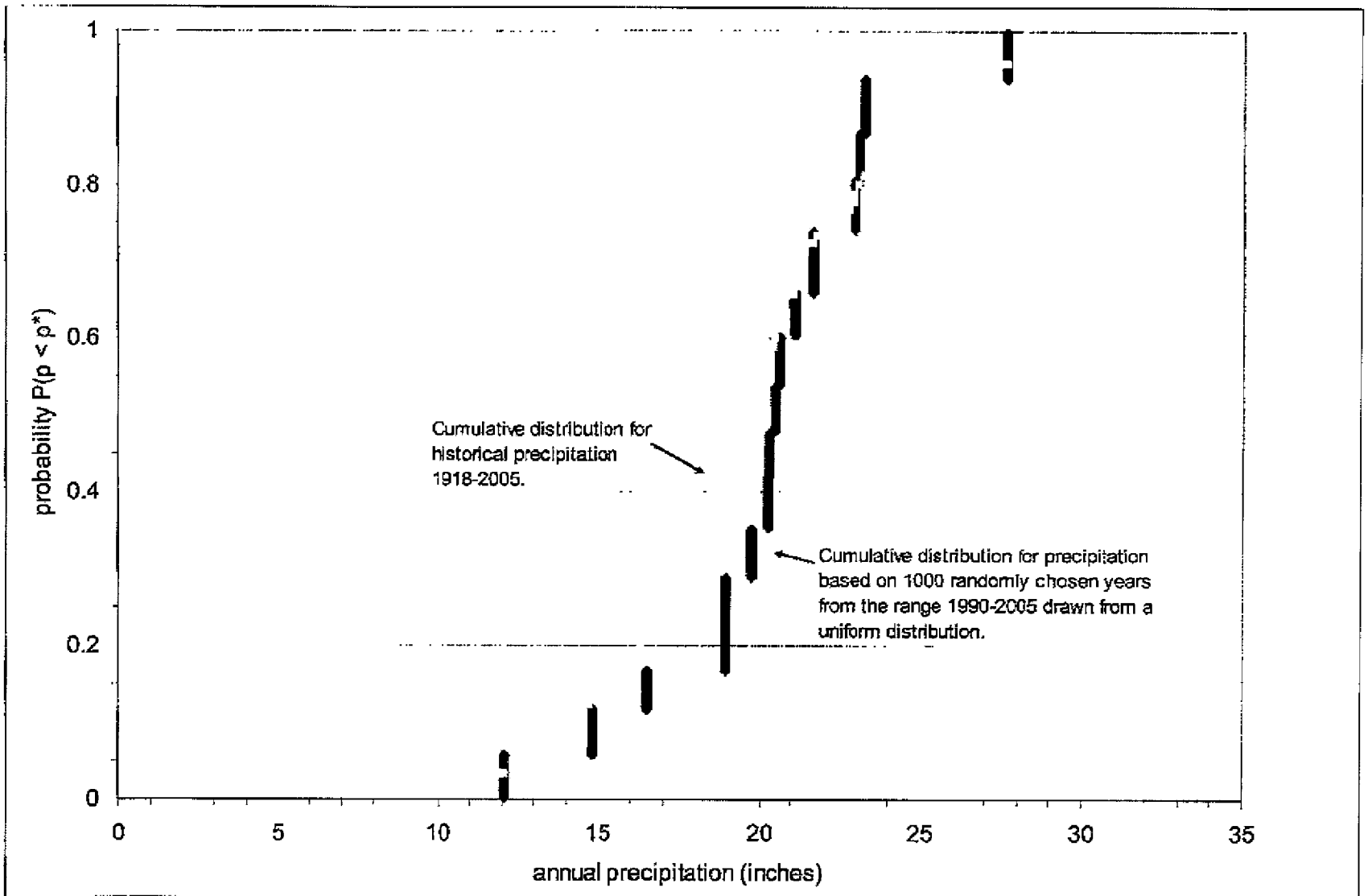

Fig. A2. Comparison of cumulative probability distribution for historical precipitation for years 1918-2006 with that for precipitation corresponding to a sequence of 1000 years randomly drawn from a uniform distribution.